

Space

An Alternate Elective after Algebra II



Henri Picciotto

MathEducation.page

Math on Another Planet

$$\begin{aligned}f &= 2e \\e &= 2d \\d &= 2f\end{aligned}$$

#1

A Long Month

Evary						
Mo	Tu	We	Th	Fr	Sa	Su
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	32
33	34	35	36	37	38	...

Isomorphism

+	d	e	f	d+e	e+f	d+f	d+e+f
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+	Mo	Tu	We	Th	Fr	Sa	Su
Mo							
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Space: topics

- Abstract algebra

- ◇ Transformational geometry

- ◇ Symmetry

- ◇ Dimension

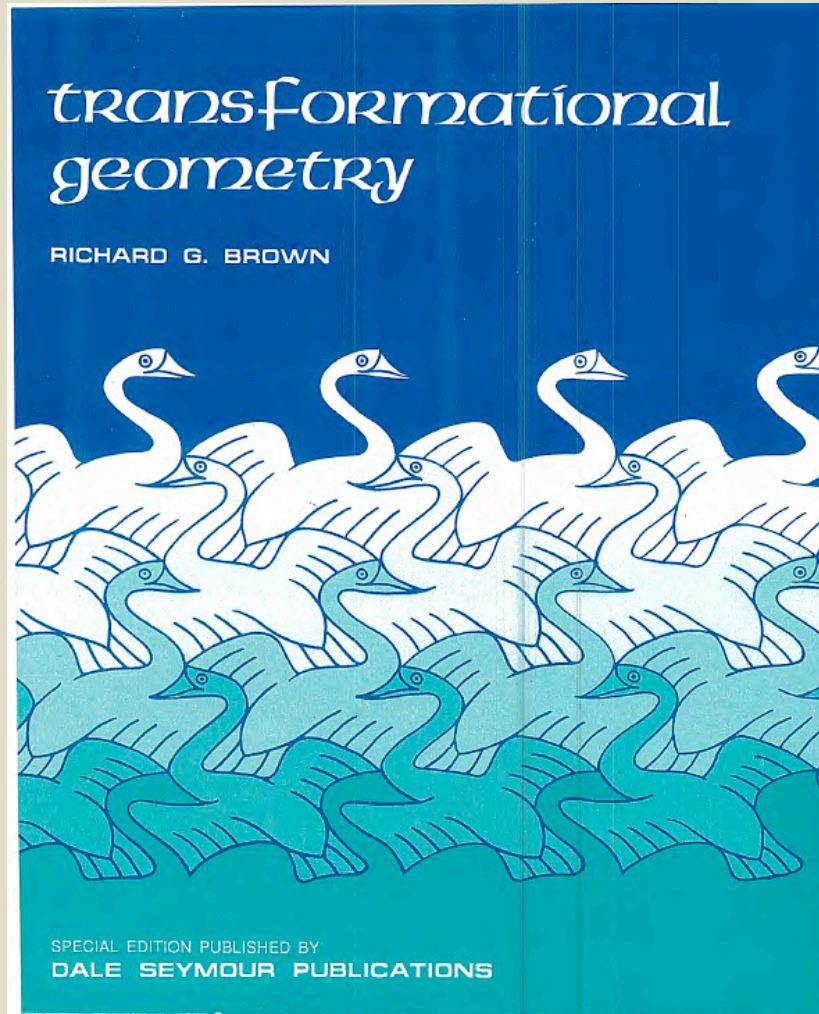
- 3D: polyhedra

- 4D: introduction

Transformations

Symmetry

Dimension



◇ Transformations: one-to-one functions
(domain, range: the whole plane)

◇ Isometries:
transformations that preserve distance

Transformations

Symmetry

Dimension

Fundamental Theorem of Isometries:
every isometry of the plane is a reflection, a rotation, a translation, or a glide reflection.

Computing transformations using complex numbers:

◇ Translation: add $a+bi$

◇ Rotation around the origin:
multiply by $\cos \theta + i \sin \theta$

◇ Rotation around (a,b) : subtract $a+bi$,
rotate around the origin, add $a+bi$

Transformations

Symmetry

Dimension

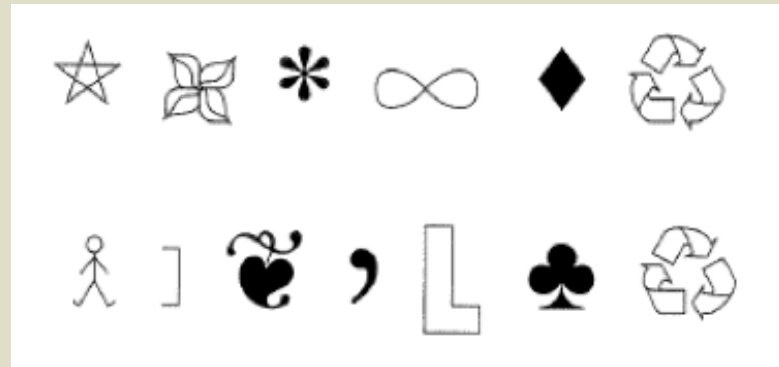
Computing transformations using matrices



Transformations

Symmetry

Dimension





Handbook of Regular Patterns by Peter Stevens

The Seven Line Symmetry Groups



I



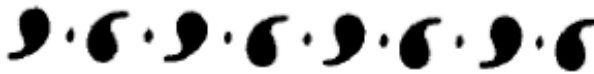
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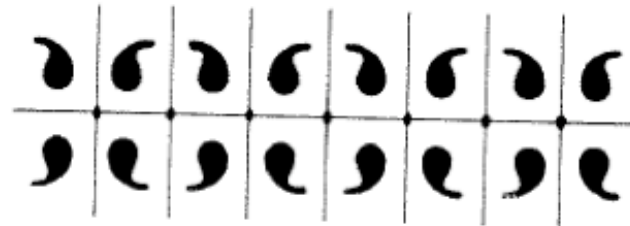
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t2

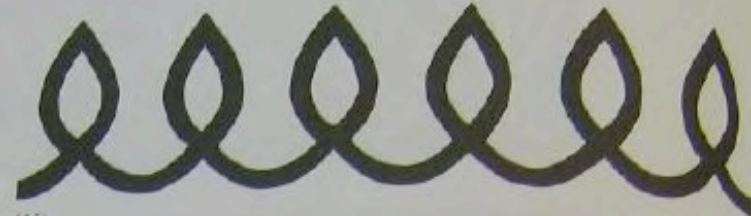


t2mg



t2mm

122
The Seven Line Group



14.5a

Figure 14.5
(a) design from Crete, first millennium B.C.
(b) Ceylonese
(c) Persian, seventeenth century
(d) French, seventeenth century



14.5b

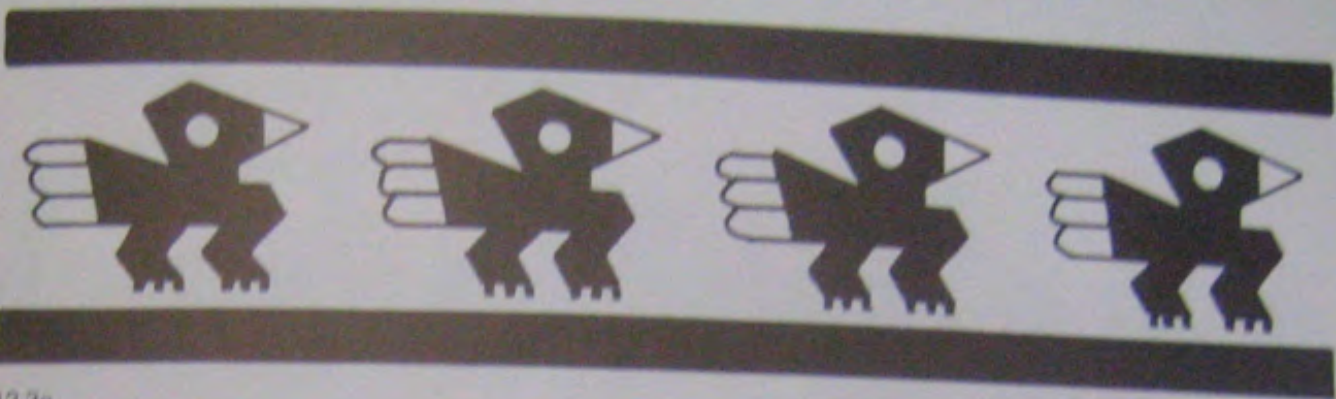


14.5c



14.5d

Figure 14.6
(a) Navaho Indian
(b) Papago Indian
(c) Pima Indian



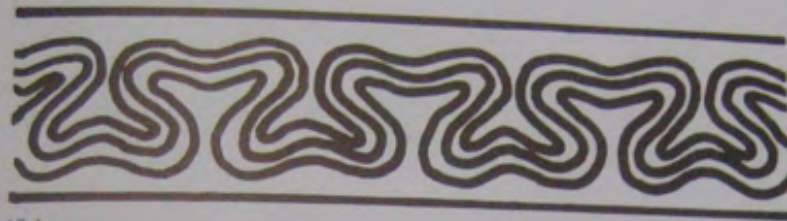
12.3a



12.3b



12.3c



17.4a



17.4b



17.4c



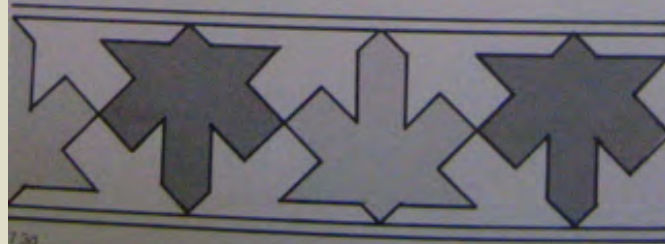
17.3a



17.3b



17.3c



17.3d

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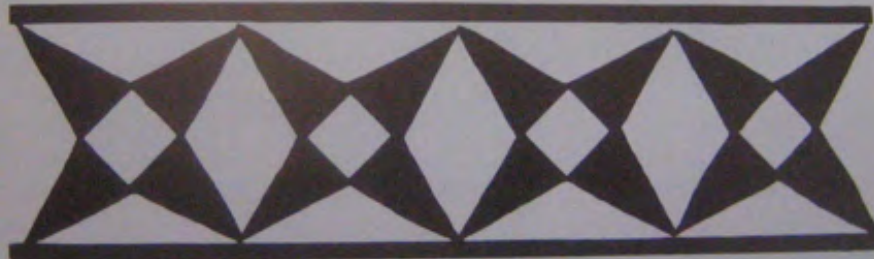
Figure 18.7
(a) Victorian ornament
(b) Arabian
(c) Pompeian mosaic
(d) Byzantine mosaic



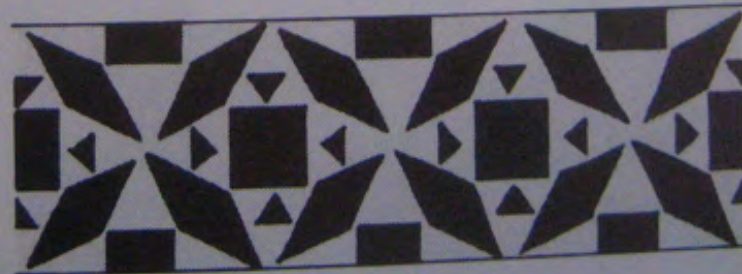
18.7a



18.7b



18.7c



18.7d



13.5e



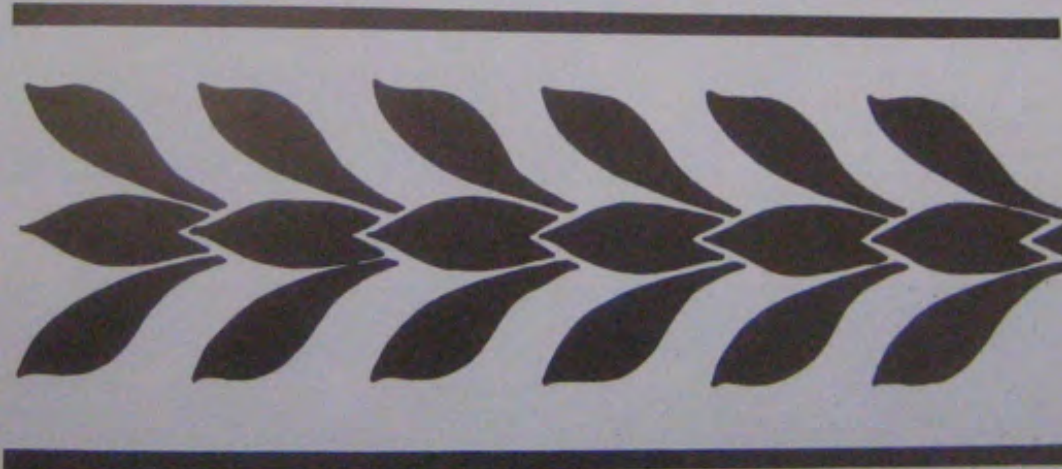
13.5f



15.4a



15.4b

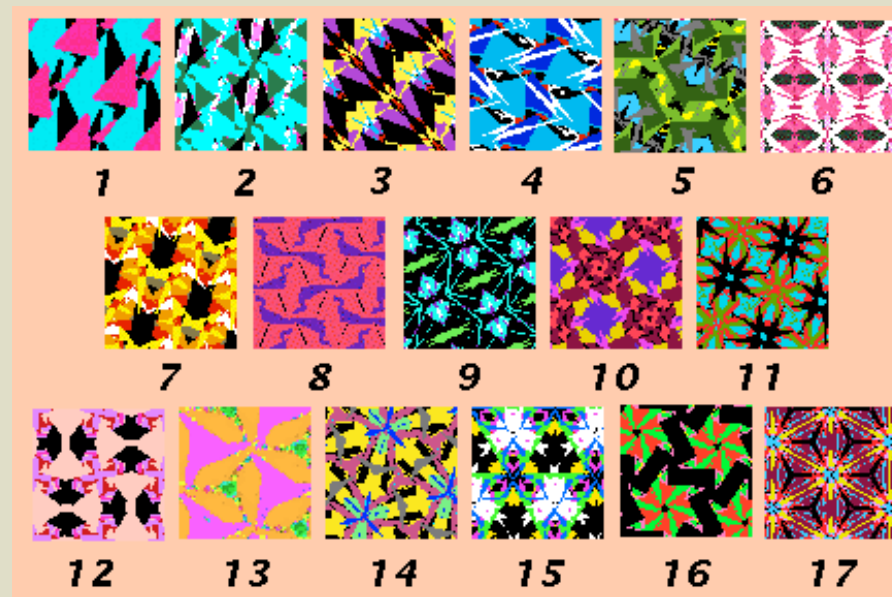


15.4c

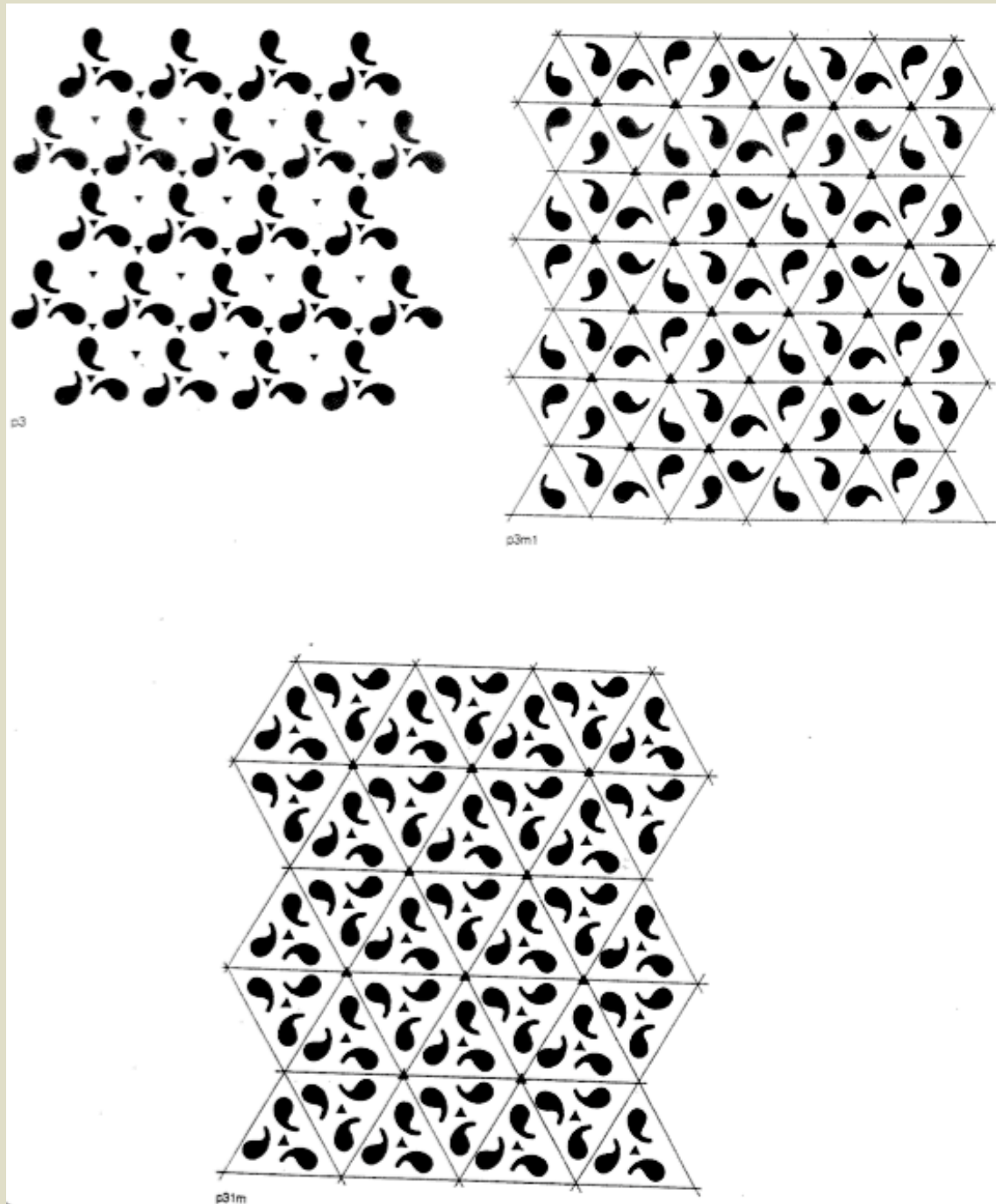


15.4d

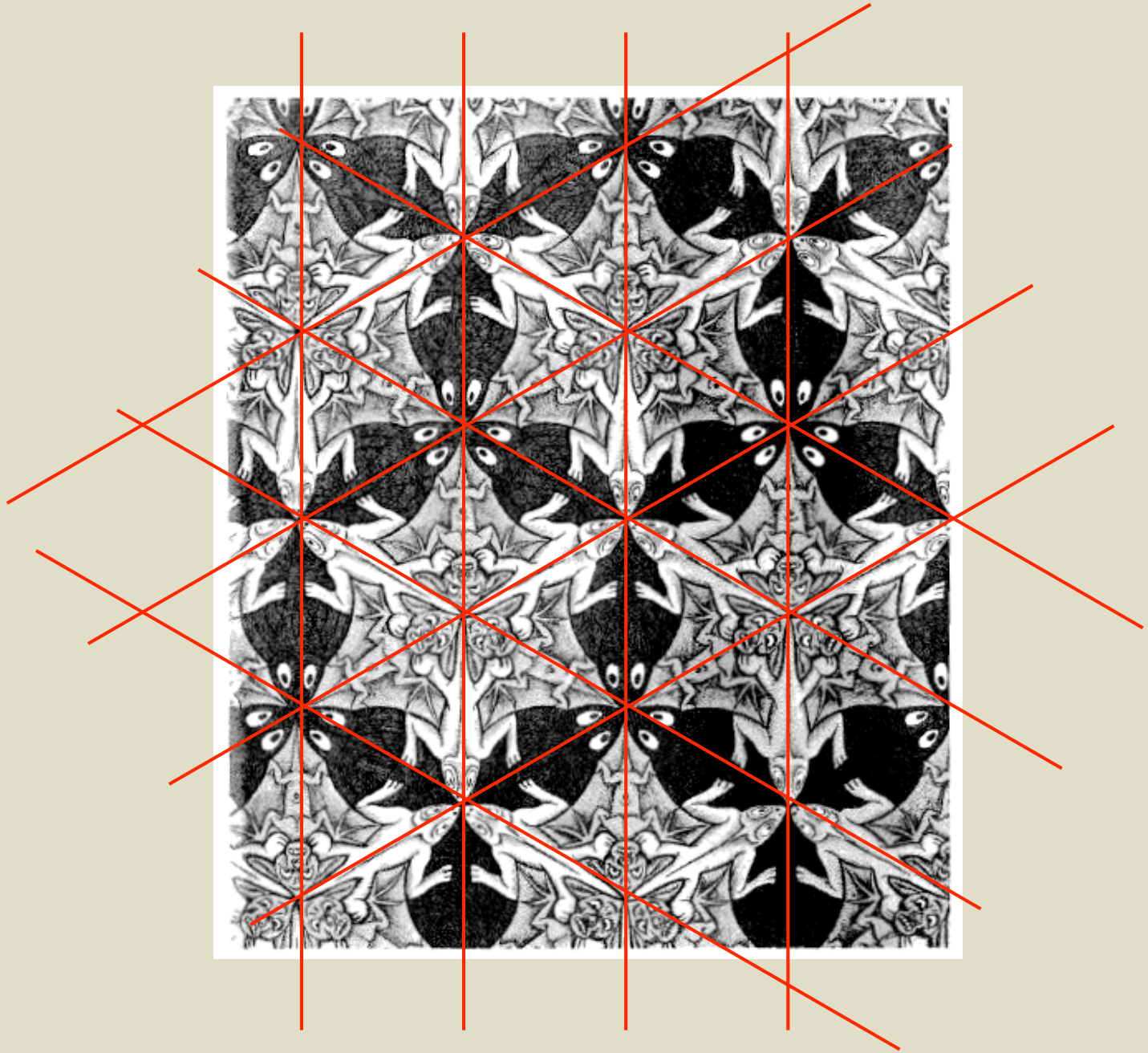
Transformations
Symmetry
Dimension



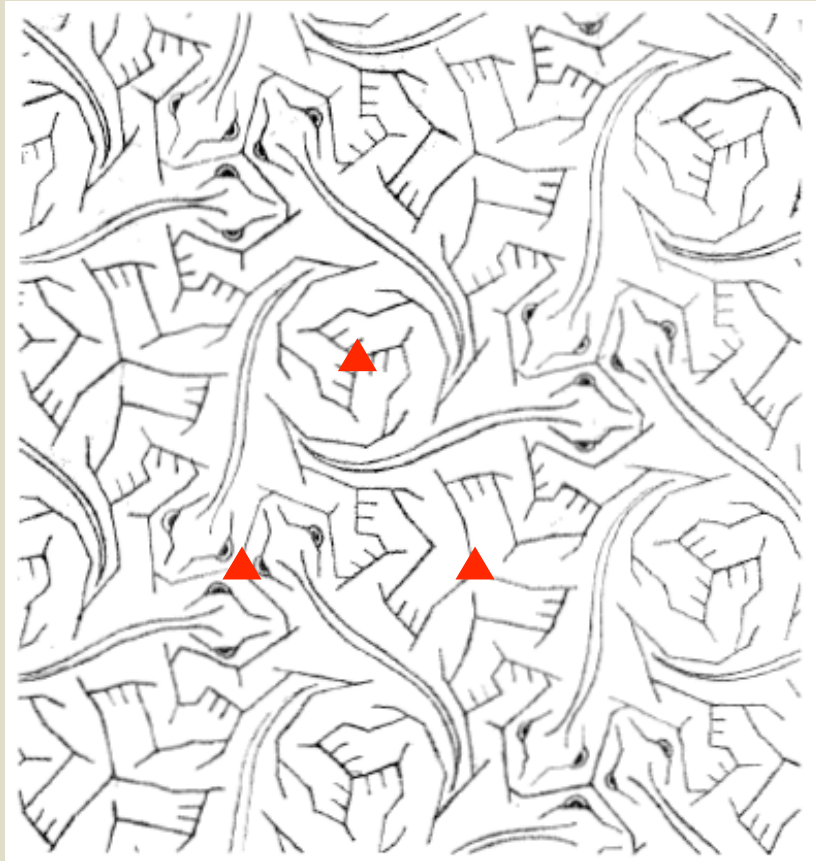
Transformations
Symmetry
Dimension



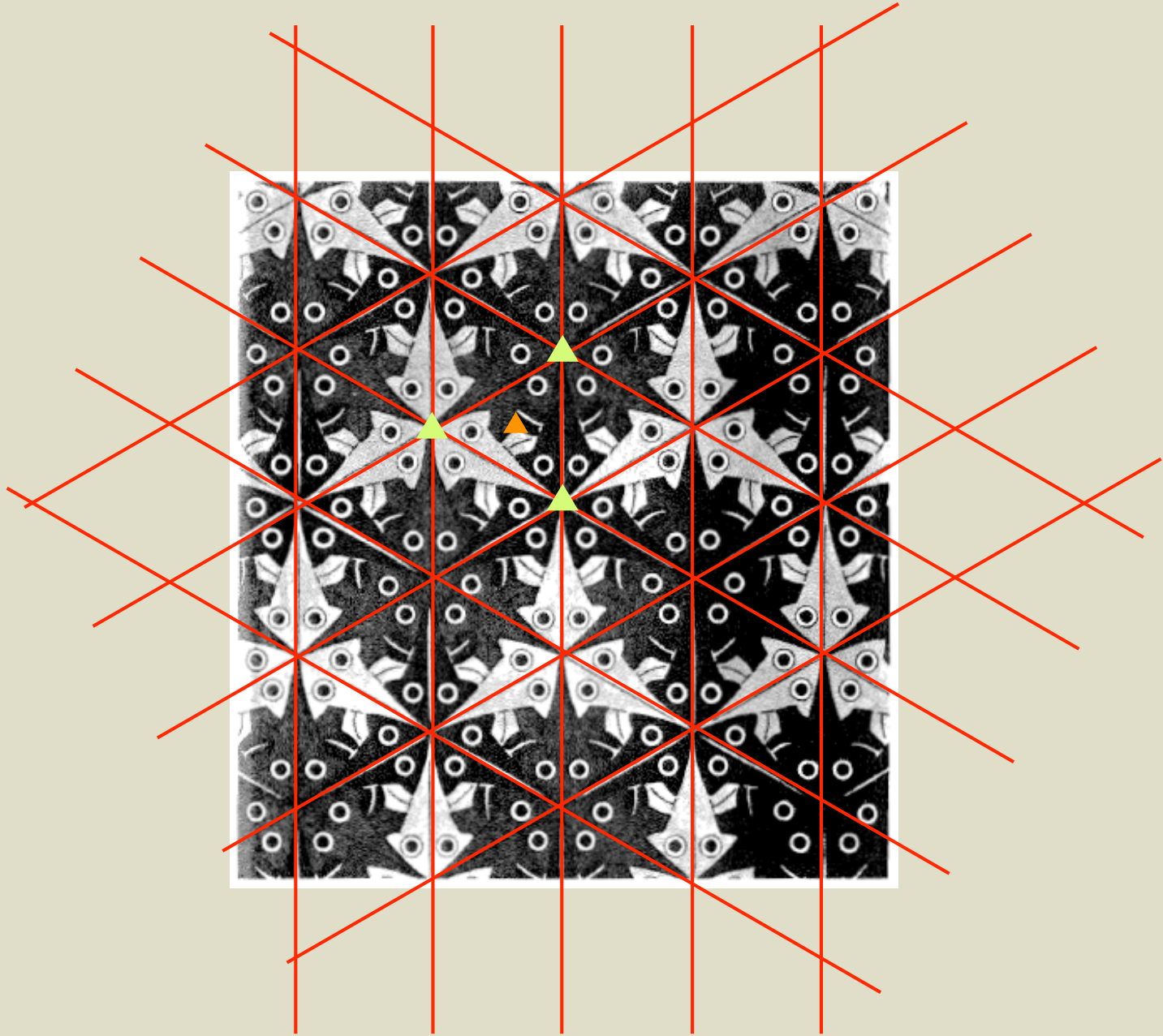
Transformations
Symmetry
Dimension

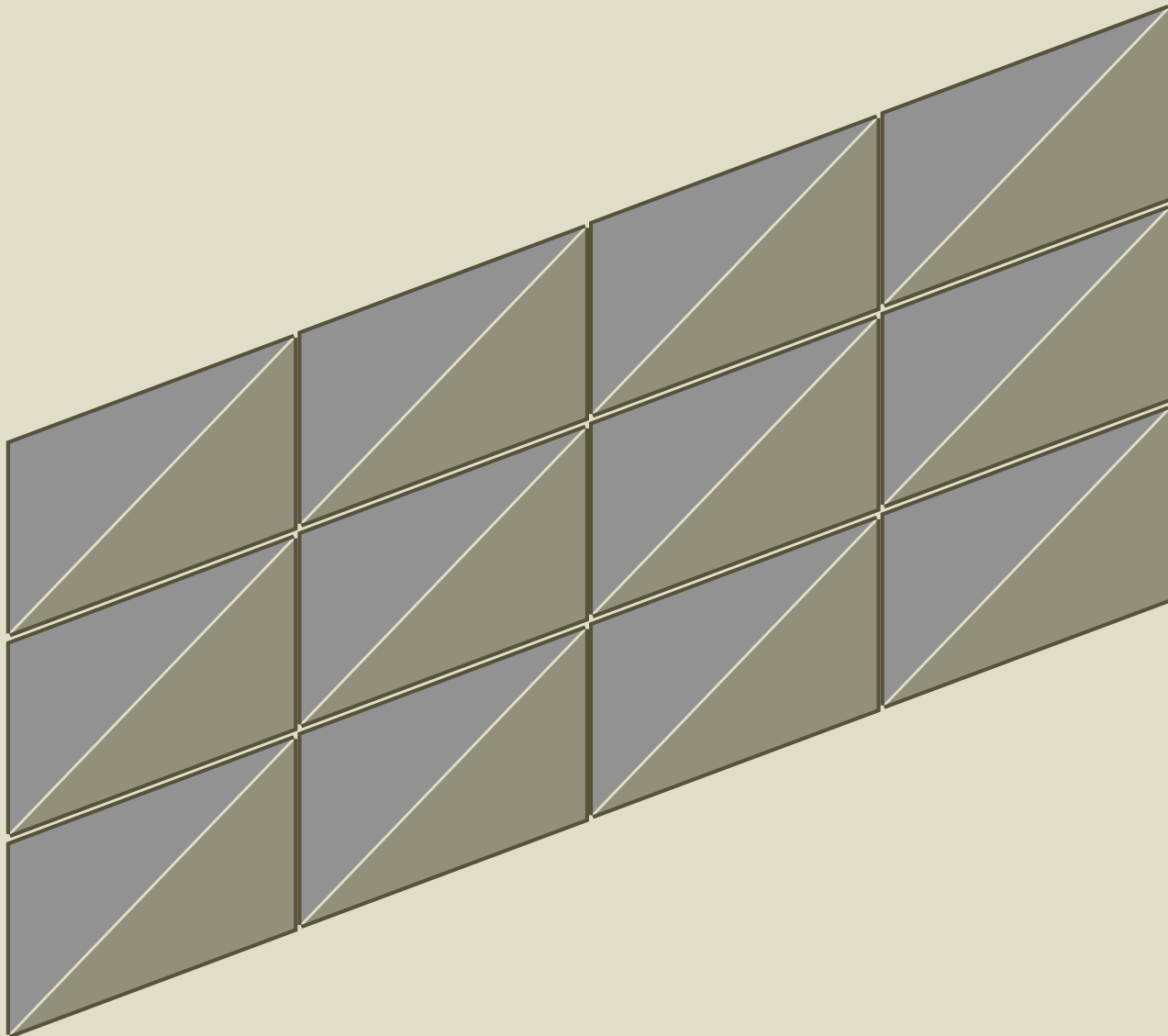


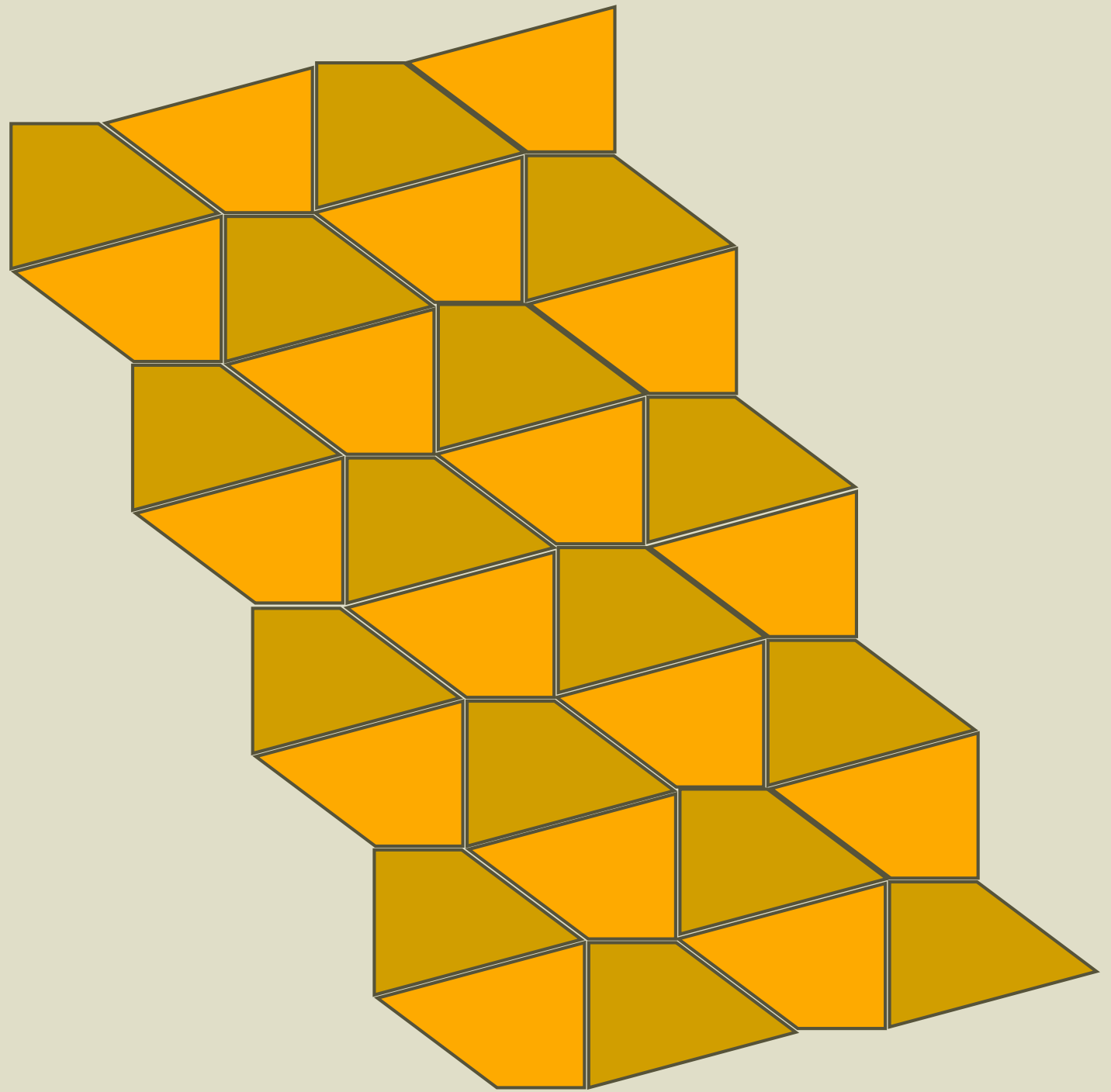
Transformations
Symmetry
Dimension



Transformations
Symmetry
Dimension







Transformations
Symmetry
Dimension



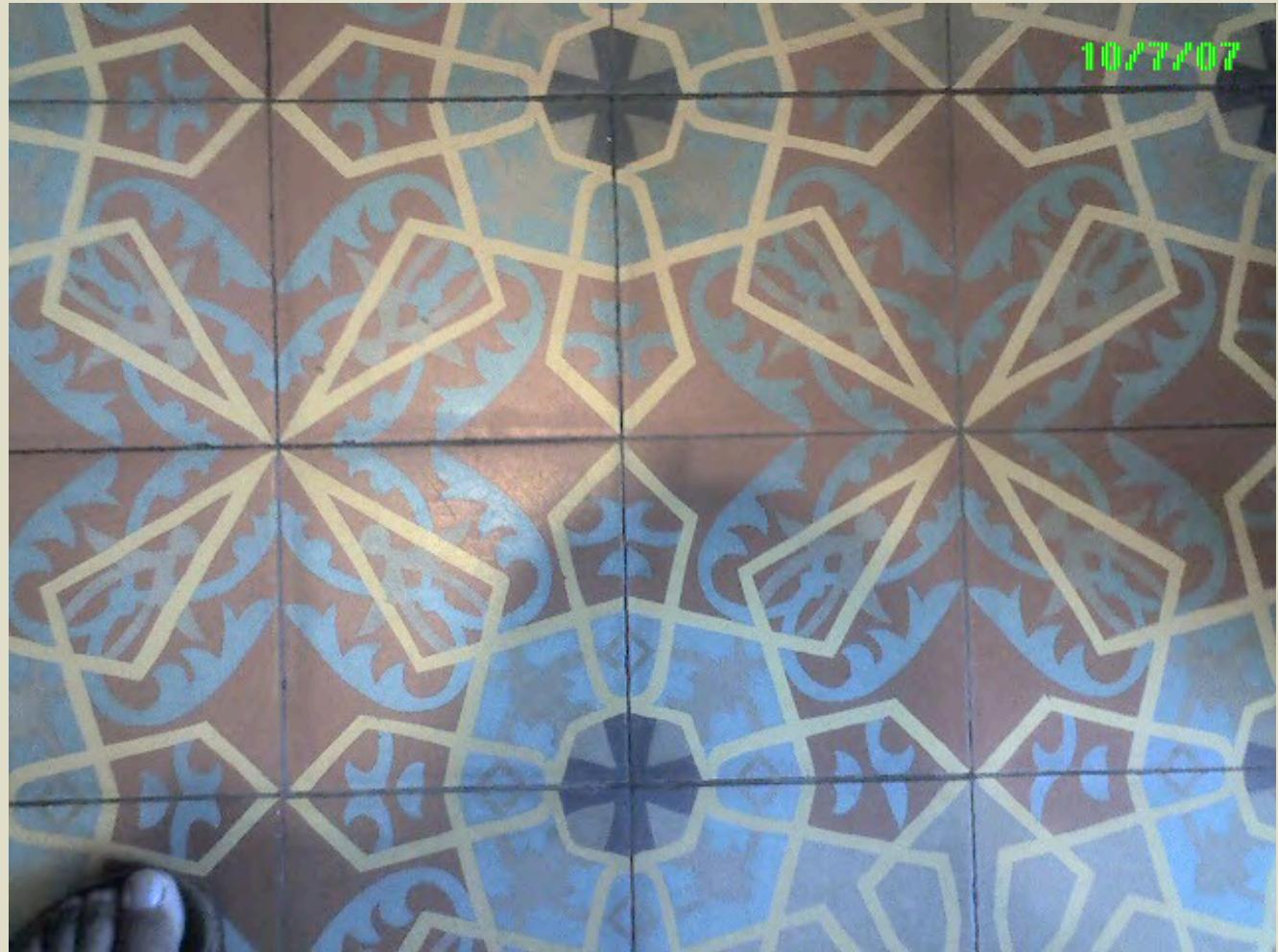
Transformations
Symmetry
Dimension



Transformations
Symmetry
Dimension



Transformations
Symmetry
Dimension



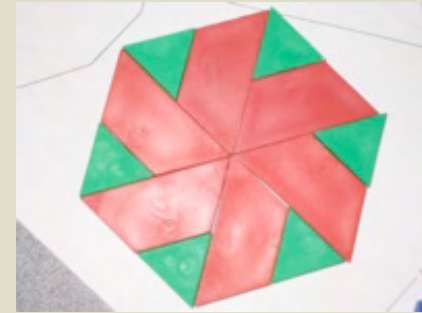
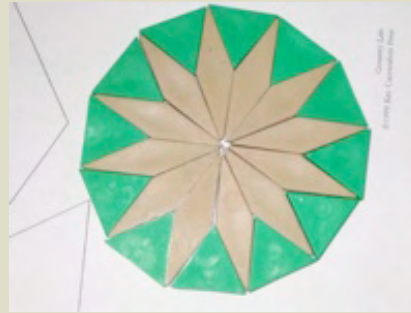
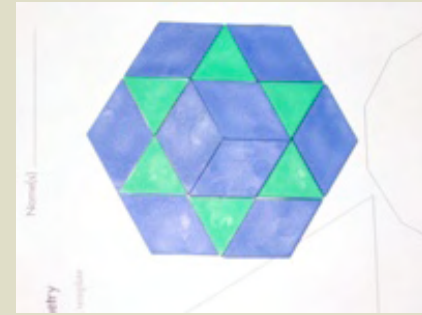
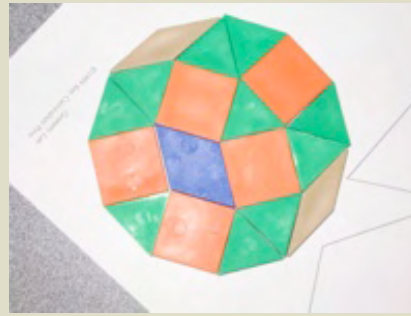
Transformations
Symmetry
Dimension



Transformations

Symmetry

Dimension



Transformations
Symmetry
Dimension

OLIVIA
LIVIA
VIA
IA
A

A whiteboard with the name 'OLIVIA' written in black marker. The letters are arranged in a square pattern: 'O' at the top-left, 'L' at the top-middle, 'I' at the top-right, 'V' at the middle-left, 'I' at the middle-middle, 'A' at the middle-right, 'I' at the bottom-left, 'V' at the bottom-middle, and 'A' at the bottom-right. A black eraser and a white marker are visible at the bottom of the board.

MIKE
IKE
KE
E

A whiteboard with the name 'MIKE' written in purple marker. The letters are arranged in a square pattern: 'M' at the top-left, 'I' at the top-middle, 'K' at the top-right, 'E' at the middle-left, 'I' at the middle-middle, 'K' at the middle-right, 'E' at the bottom-left, 'I' at the bottom-middle, and 'K' at the bottom-right.

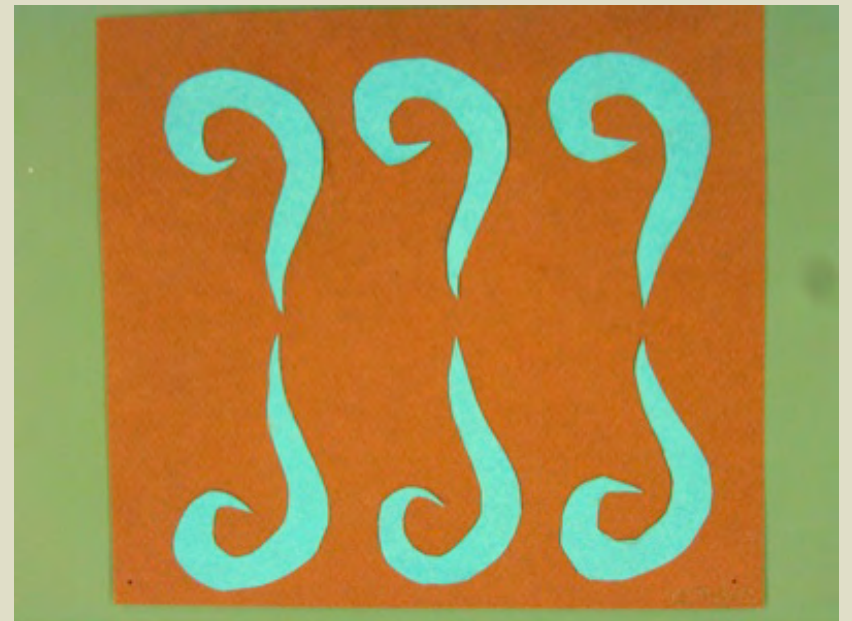
SIS

A whiteboard with the word 'SIS' written in purple cursive marker. The letters are connected and stylized.

Emily

A whiteboard with the name 'Emily' written in purple cursive marker. The letters are connected and stylized.

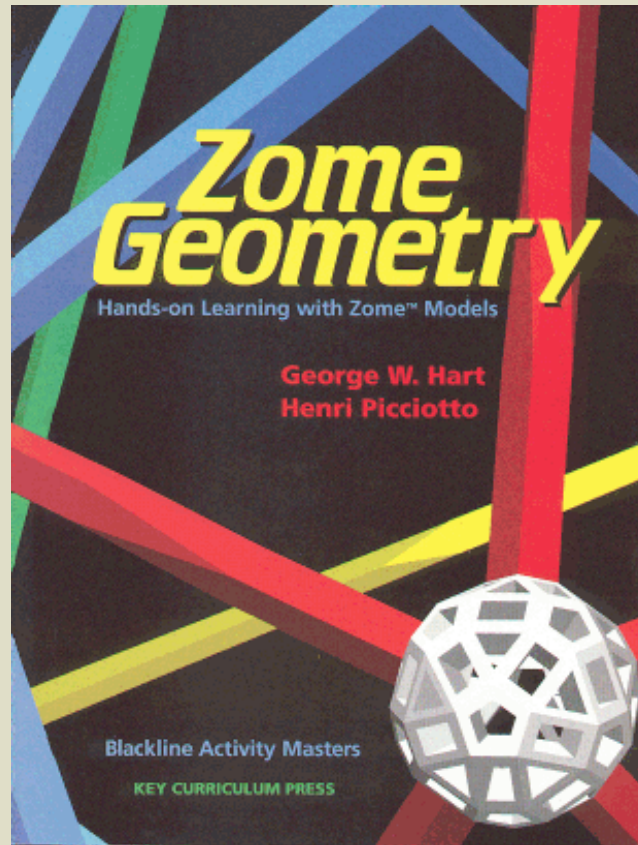
Transformations
Symmetry
Dimension



Transformations
Symmetry
Dimension



Transformations
Symmetry
Dimension: 3D



Transformations

Symmetry

Dimension: 3D

- ◇ Platonic and Archimedean polyhedra
- ◇ Duality
- ◇ Euler's and Descartes' theorems
- ◇ Review of geometry and trigonometry

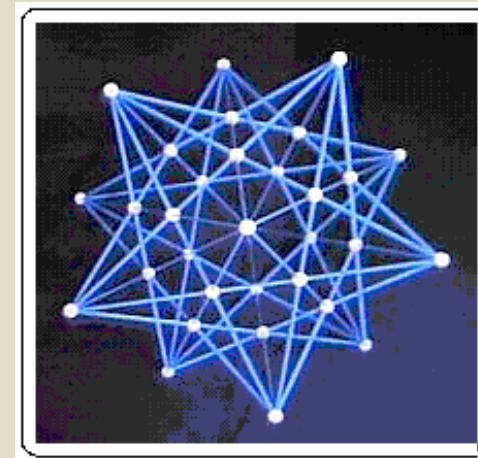
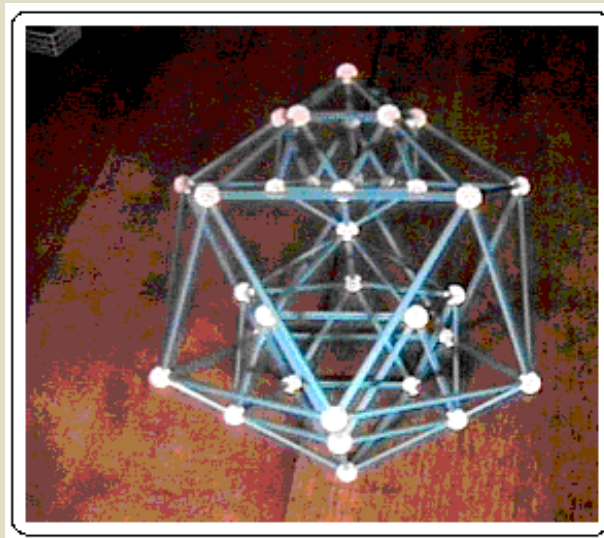
Transformations

Symmetry

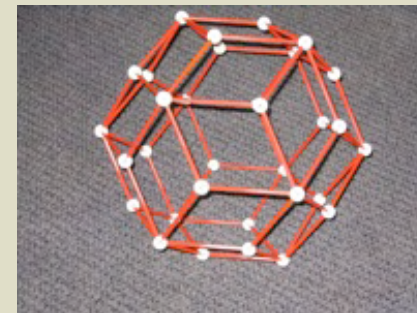
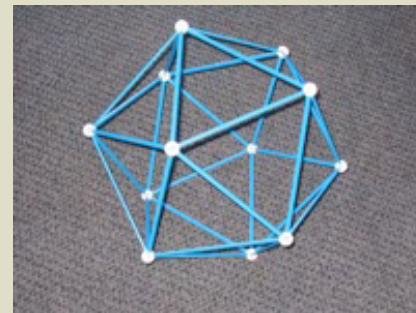
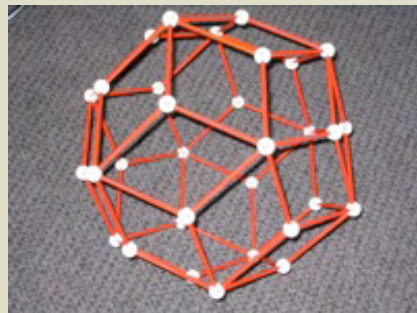
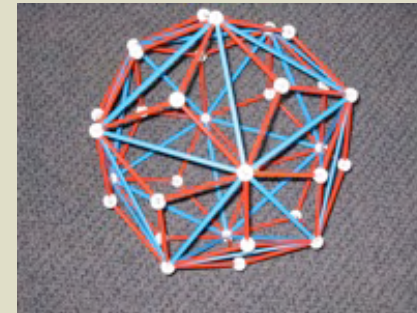
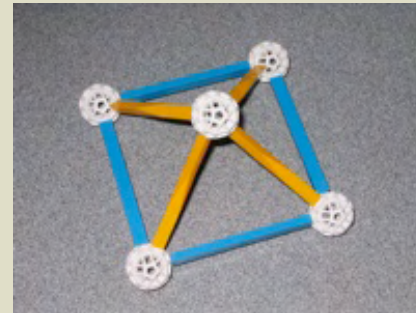
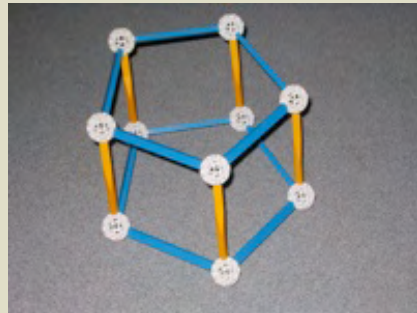
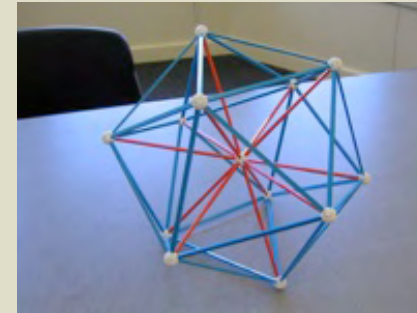
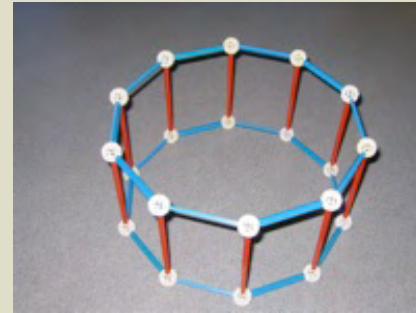
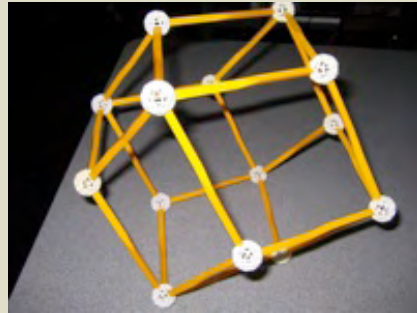
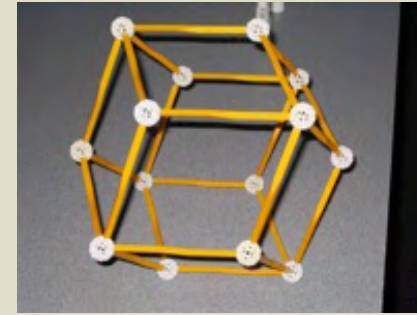
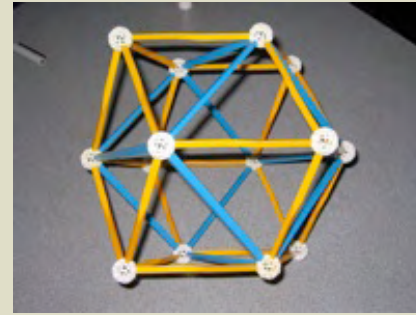
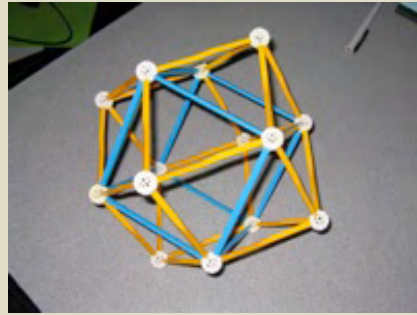
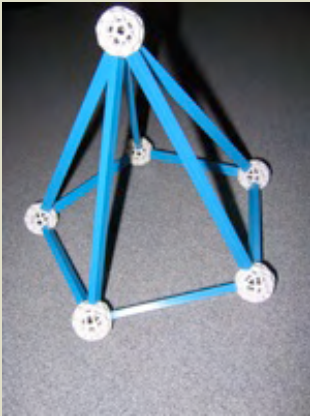
Dimension: 3D

The chief reason for studying regular polyhedra is still the same as in the time of the Pythagoreans, namely, that their symmetrical shapes appeal to one's artistic sense.

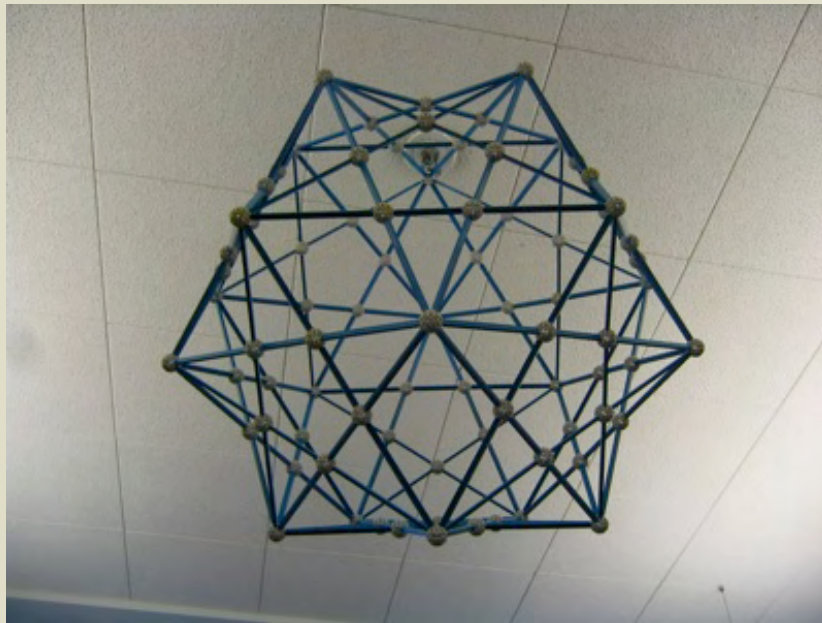
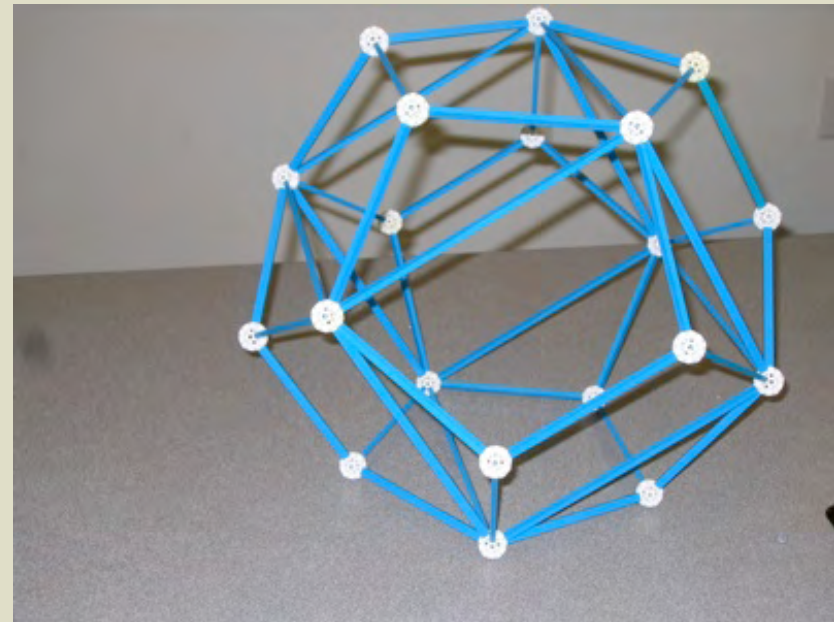
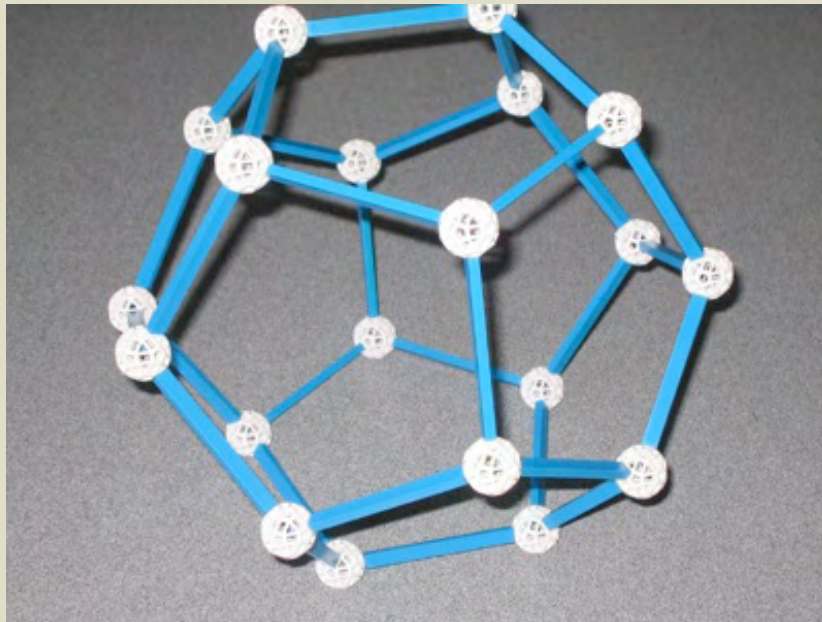
---H.S.M. Coxeter



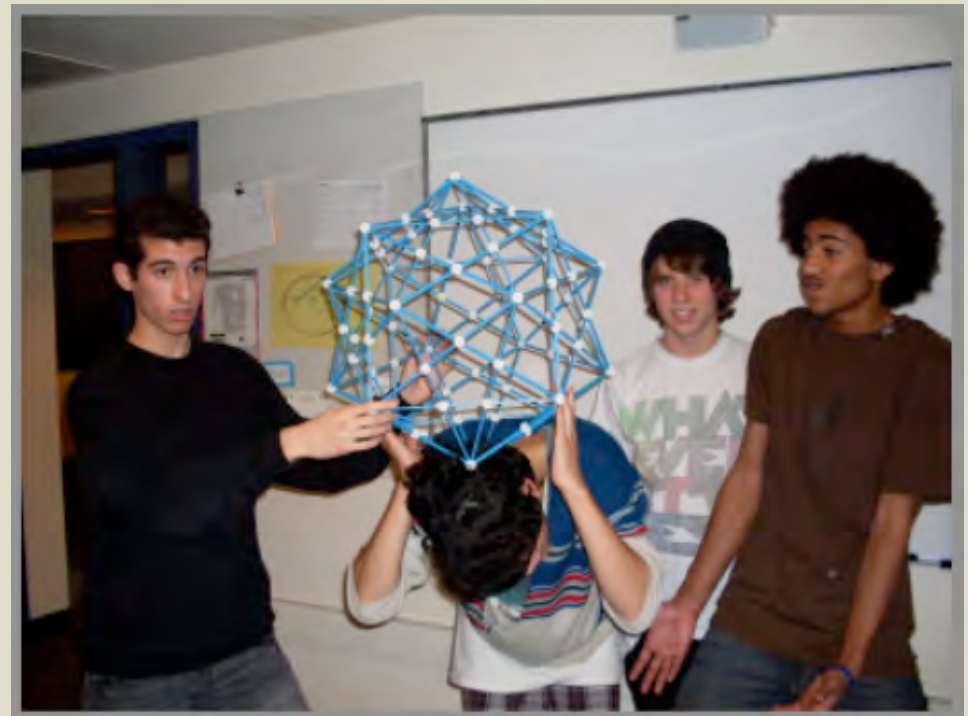
Transformations
Symmetry
Dimension: 3D



Transformations
Symmetry
Dimension: 3D



Transformations
Symmetry
Dimension: 3D



Transformations
Symmetry
Dimension: 3D

CABRI® 3D v2 CABRILOG

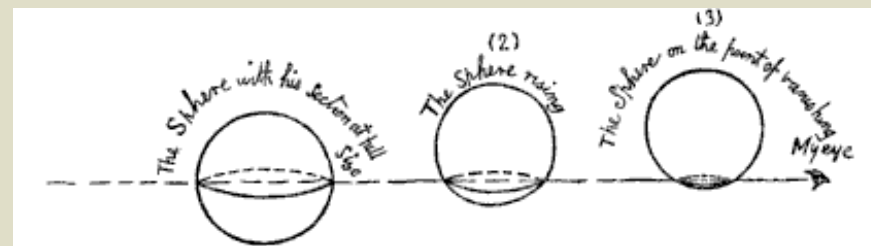
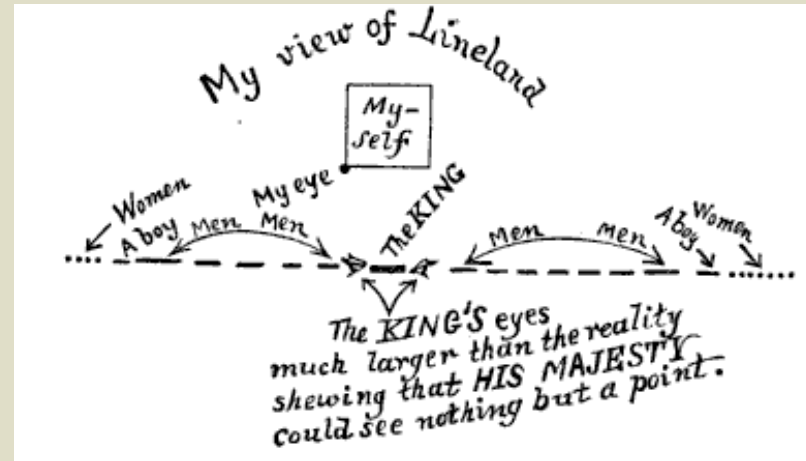
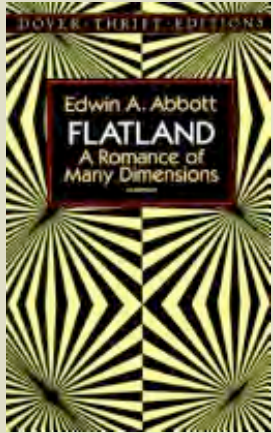
$$V = \frac{1}{3} \pi R^2 h$$
$$V = a^3$$

Cabri 3D 2.1.1 -- Copyright © 2004-2007 Cabrilog
Design, architecture: Eric Bainville, Jean-Marie Laborde
Development and Software Quality: Cabrilog

BETT
AWARDS 2007
WINNER

2007
MINISTÈRE DE
L'ÉDUCATION NATIONALE
reconnu d'intérêt
pédagogique

Transformations
Symmetry
Dimension: 4D





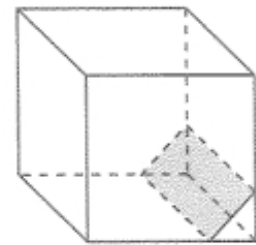
LAB 6.5 Slicing a Cube

Name(s) _____

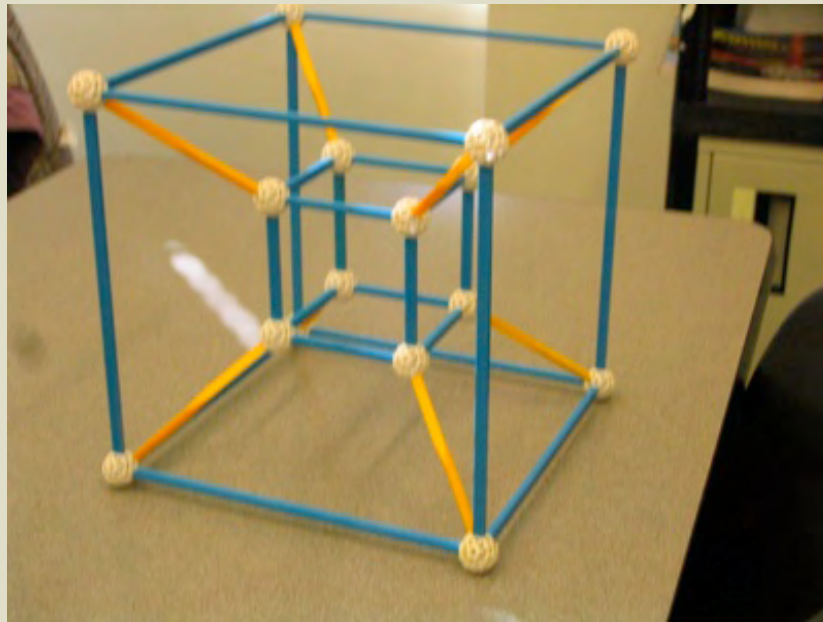
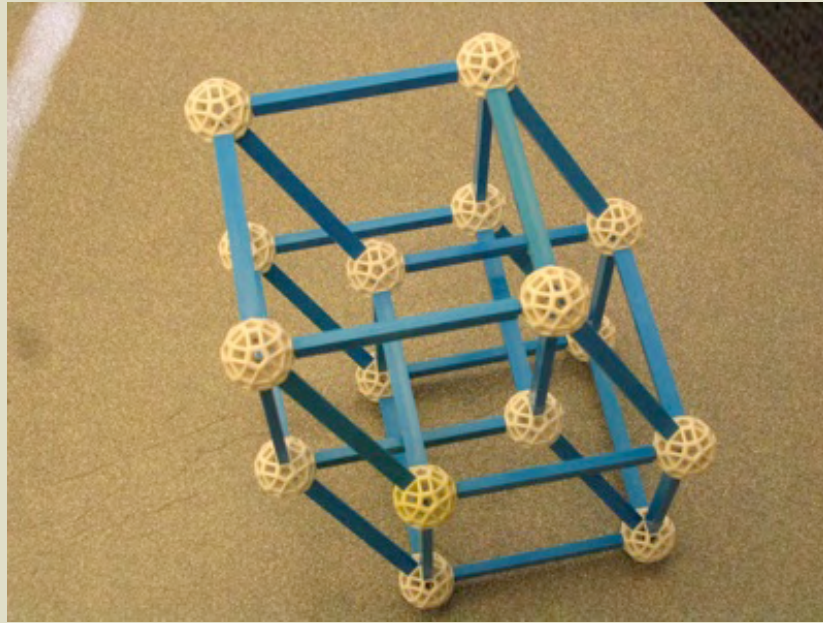
- **Equipment:** Transparency, stiff paper, scissors, adhesive tape

Imagine that you slice a cube in a direction parallel to one of the faces. The shape of the slice will be a square. Now imagine that you slice a cube parallel to one of the bottom edges, but tilted from the horizontal. The shape of the slice will be a rectangle.

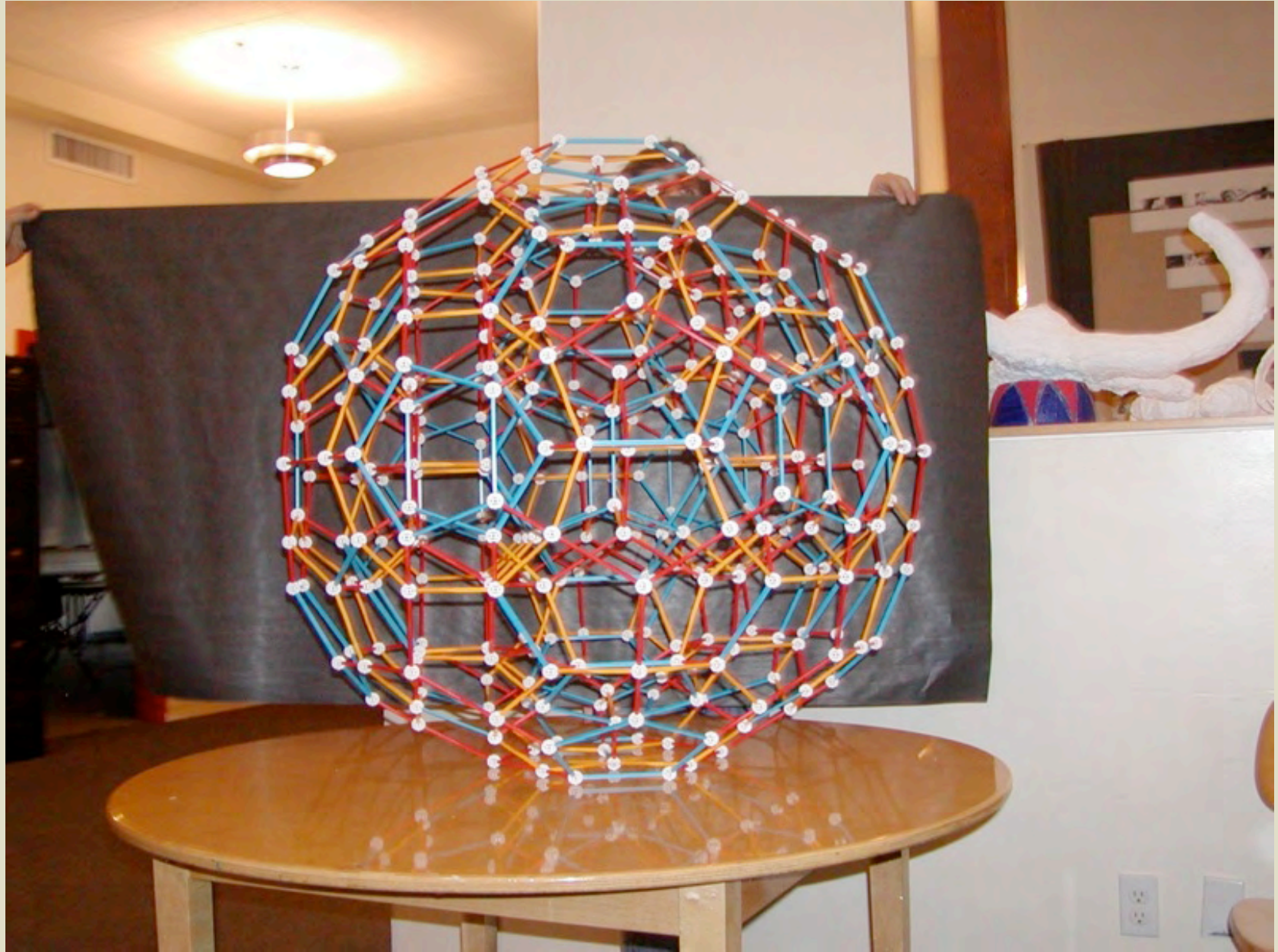
What shapes are possible for a slice? To investigate this, you will use a hollow transparent cube and stiff paper to simulate the slices.



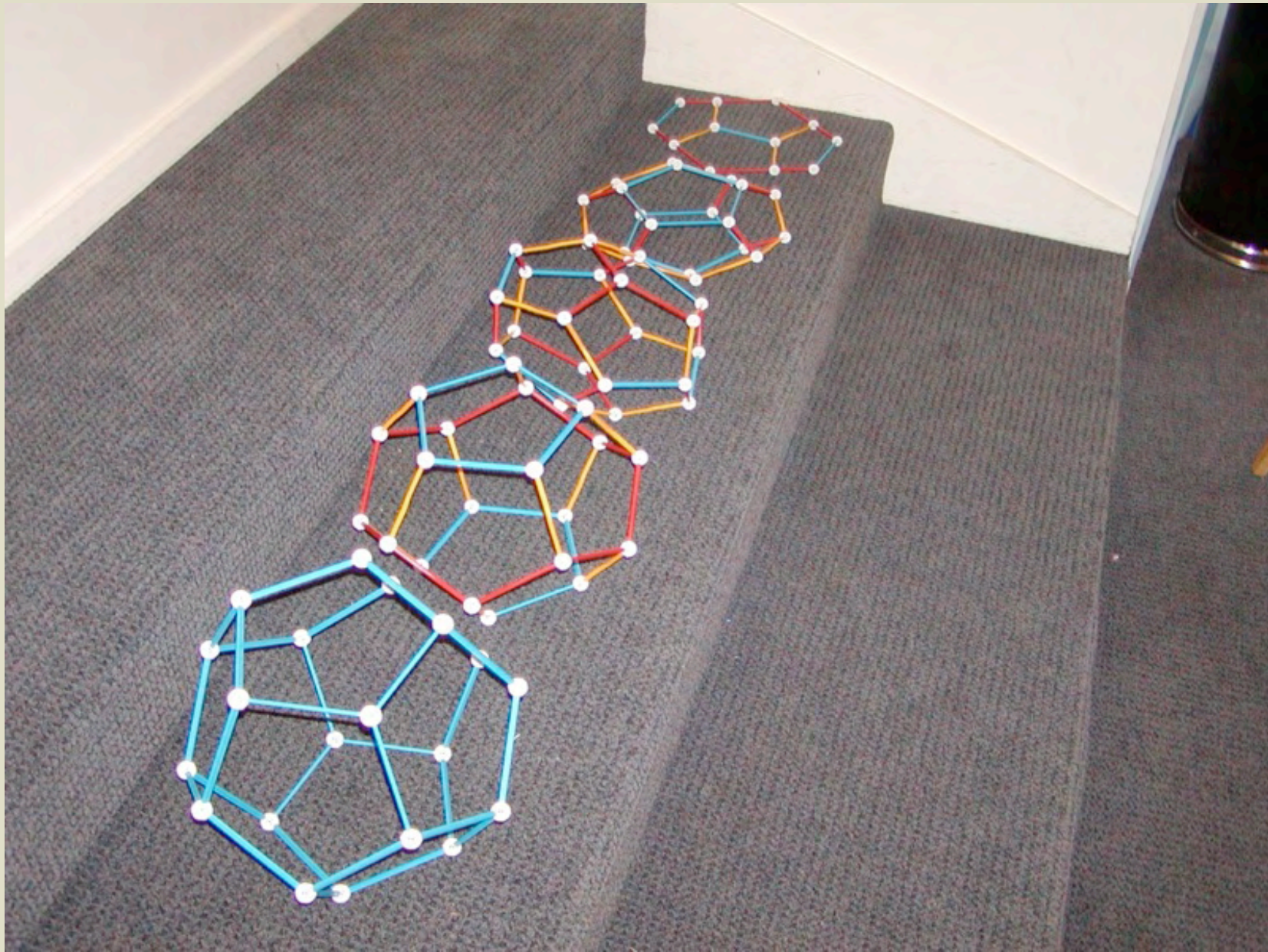
Transformations
Symmetry
Dimension: 4D



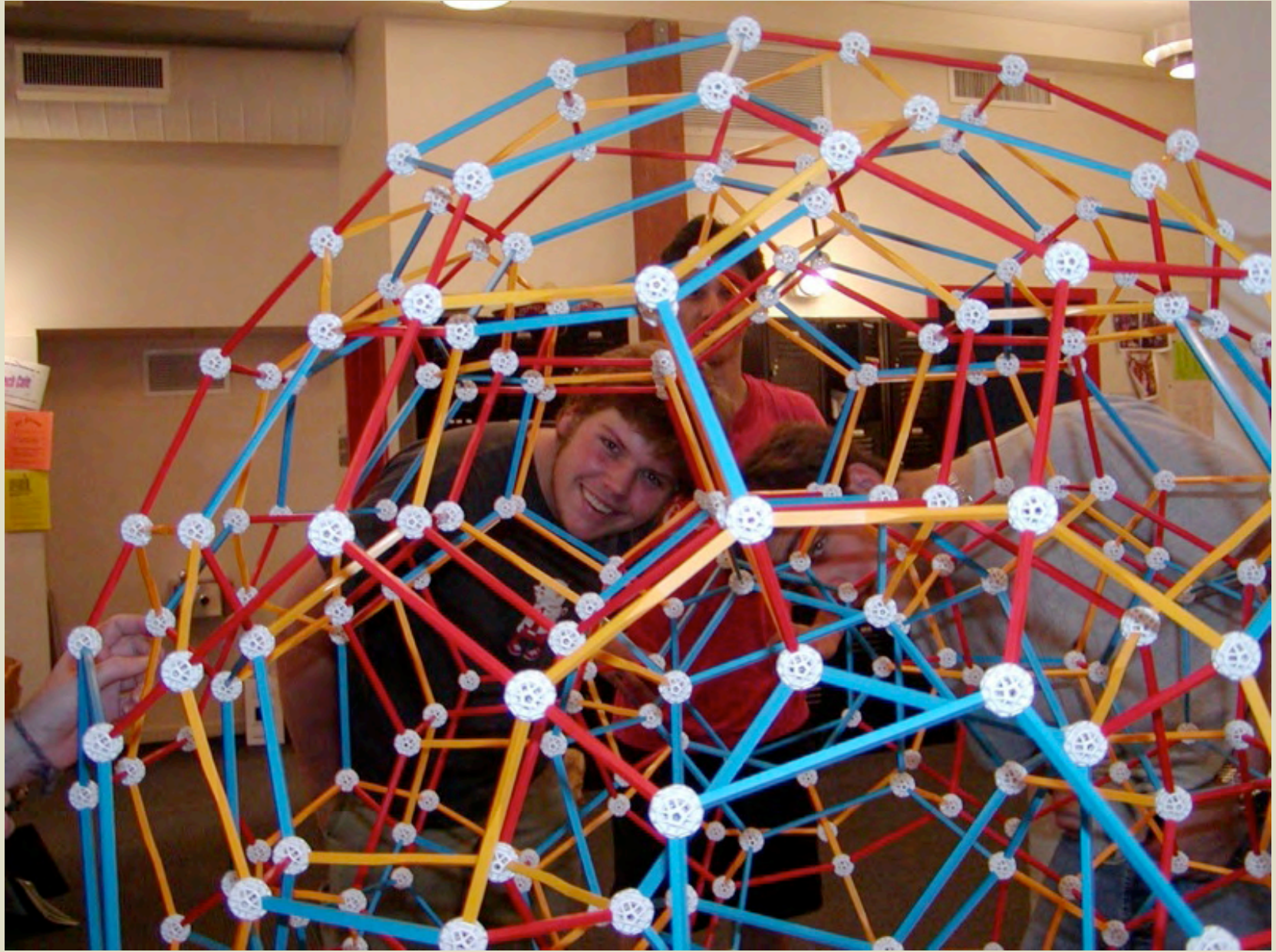
Transformations
Symmetry
Dimension: 4D



Transformations
Symmetry
Dimension: 4D



Transformations
Symmetry
Dimension: 4D



Space

An Alternate Elective after Algebra II



Henri Picciotto

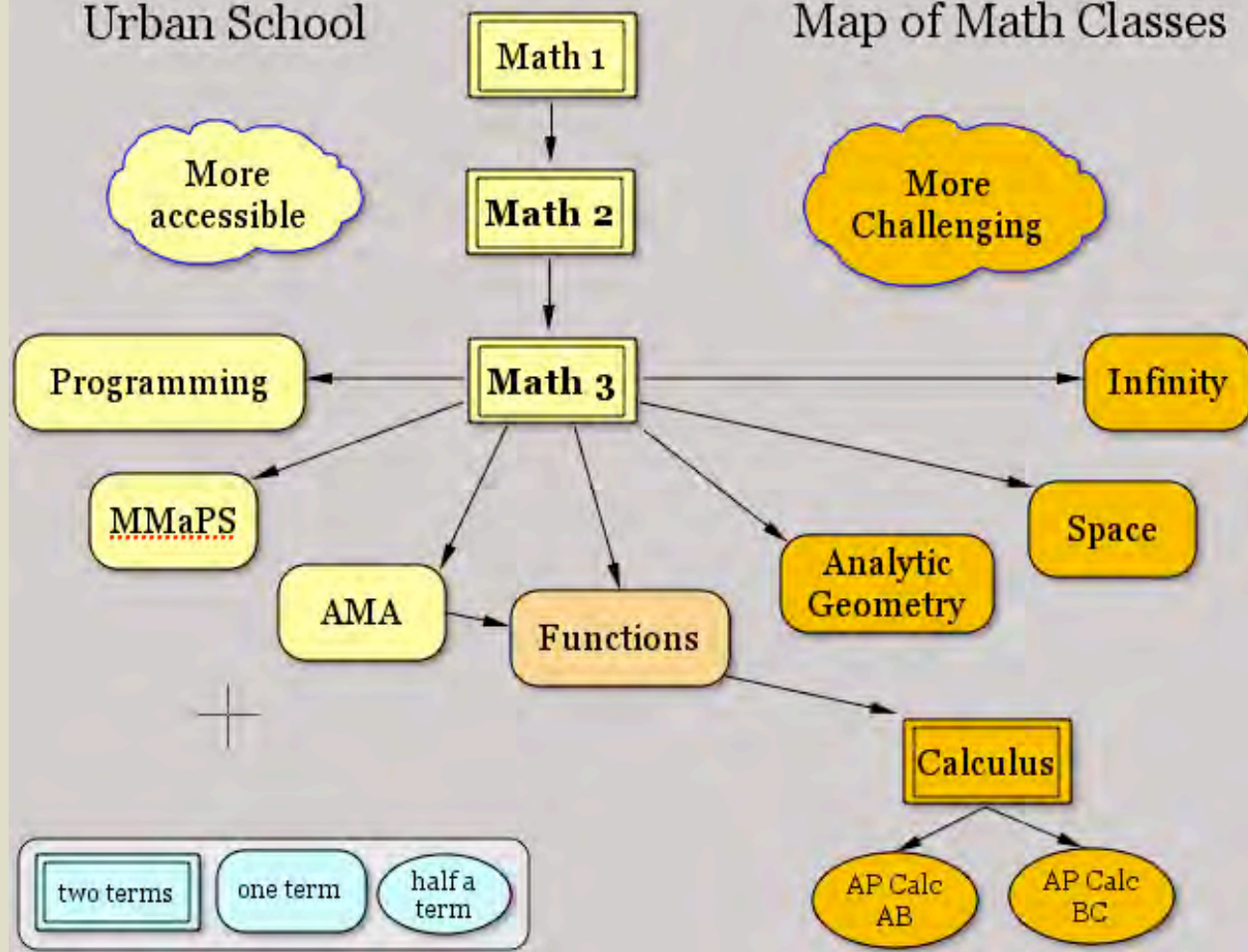
The Urban School of San Francisco

math-ed@picciotto.org

www.picciotto.org/math-ed

Urban School

Map of Math Classes



Space overview

Who takes the class

Topics

Juniors, before
Calculus

Review

Seniors, instead of
or in addition to
Calculus

Resources

Computer tools

Space overview

Who takes the class

Topics

Abstract algebra

Transformations

Review

Symmetry

Resources

Dimension (3D, 4D)

Computer tools

Space overview

Who takes the class

Topics

Review

Resources

Computer tools

Algebra

Geometry

Trigonometry

Space overview

Who takes the class

Transformational Geometry
by Richard Brown

Topics

Algebra: Themes, Tools, Concepts
by Anita Wah and Henri Picciotto

Review

Geometry Labs by Henri Picciotto

Resources

Handbook of Regular Patterns
by Peter Stevens

Computer tools

Zome Geometry
by George Hart and Henri Picciotto

Flatland by Edwin Abbott

Space overview

Who takes the class

Topics

Cabri II+

Review

Cabri 3D

Resources

(vZome)

Computer tools

Space

An Alternate Elective after Algebra II



Henri Picciotto

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math-ed@picciotto.org

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