Geometric Puzzles: Tiles and Rep-Tiles

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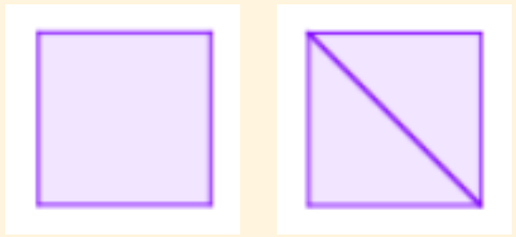
Tangrams

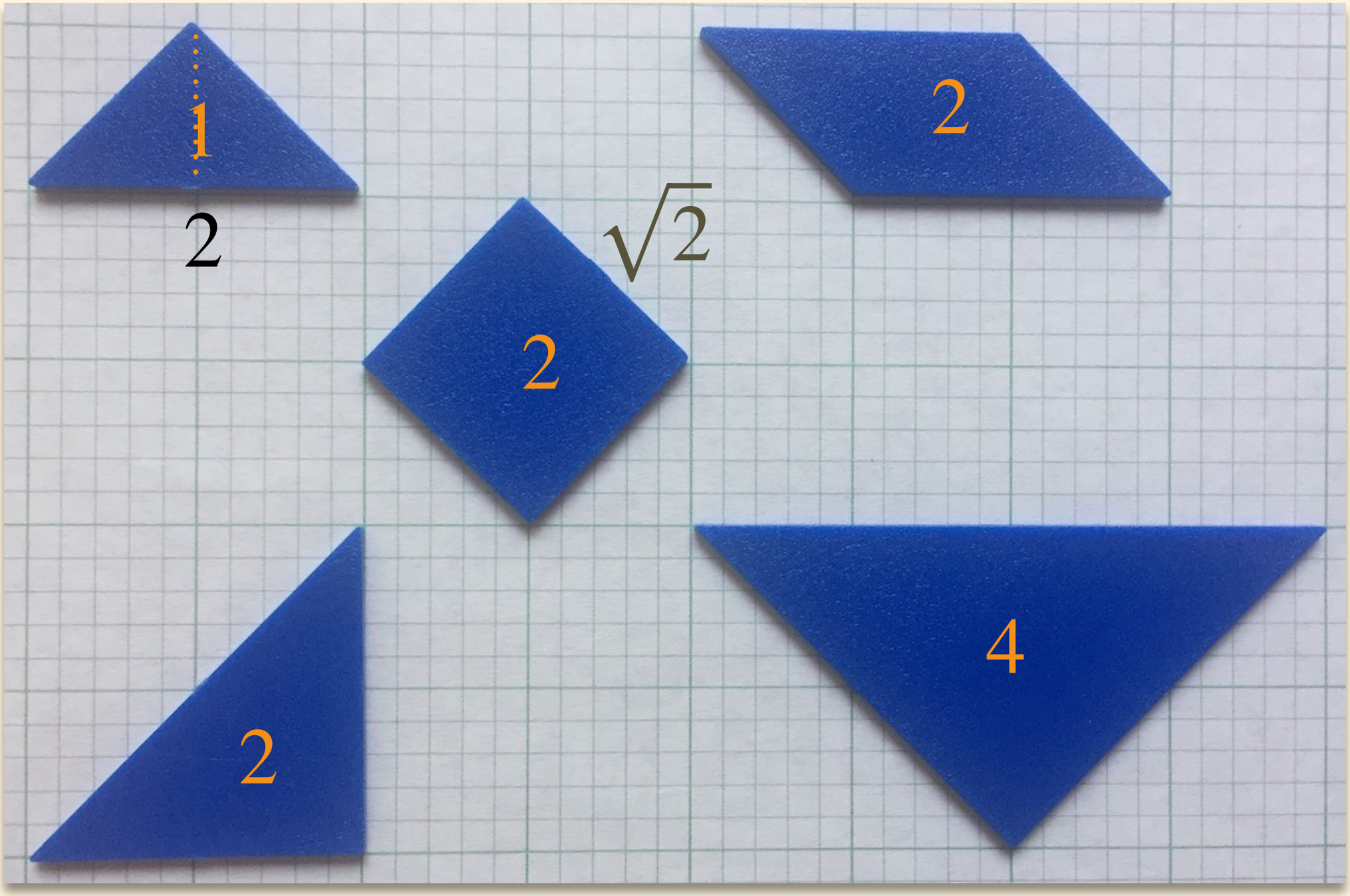
Make a square, using 1 to 7 pieces.

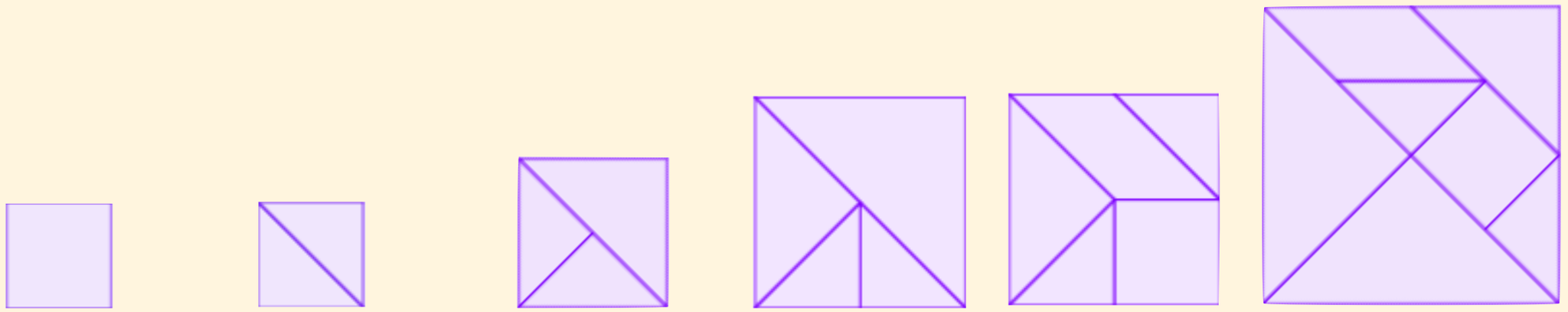
What squares are possible?



Tangram Measurements

(inches and square inches)





Pieces	1	2	3	4	5	7
Area	2	2	4	8	8	16
Side	$\sqrt{2}$	$\sqrt{2}$	2	$2\sqrt{2}$	$2\sqrt{2}$	4

$$\sqrt{8} = 2\sqrt{2}$$

A 6-piece square is impossible

◊ Total tangram area: 16 in^2

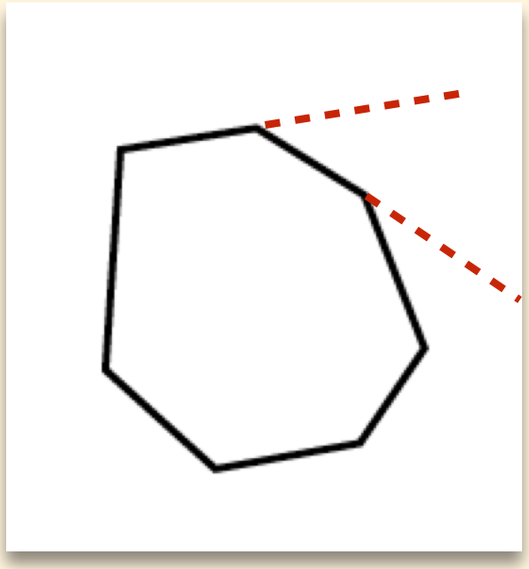
◊ Individual pieces: 1, 2, or 4 in^2

◊ 6-piece area: 15, 14, or 12 in^2

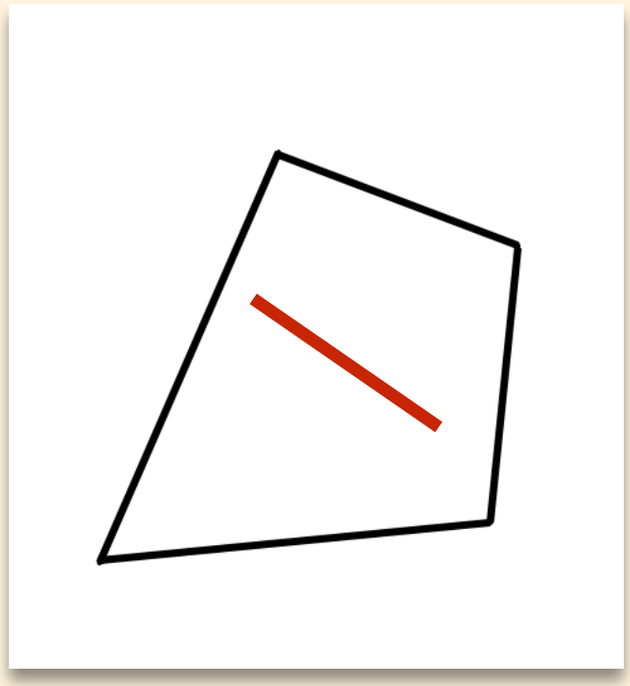
◊ Side of 6-piece square?

Convex Polygons

All “turn angles” turn in the same direction

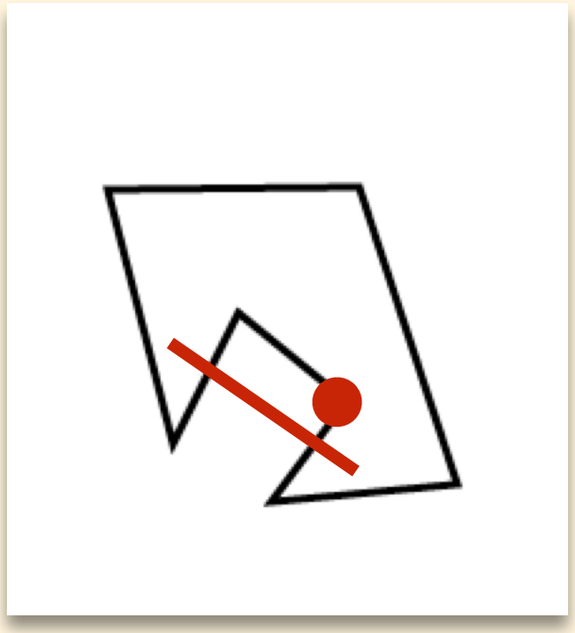


yes



yes

All interior angles < 180°

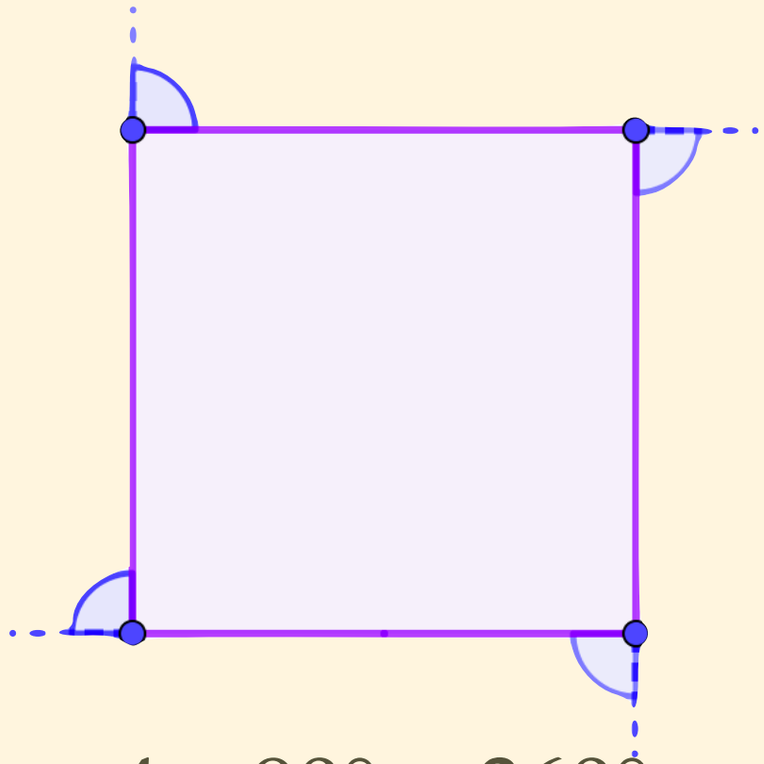


no

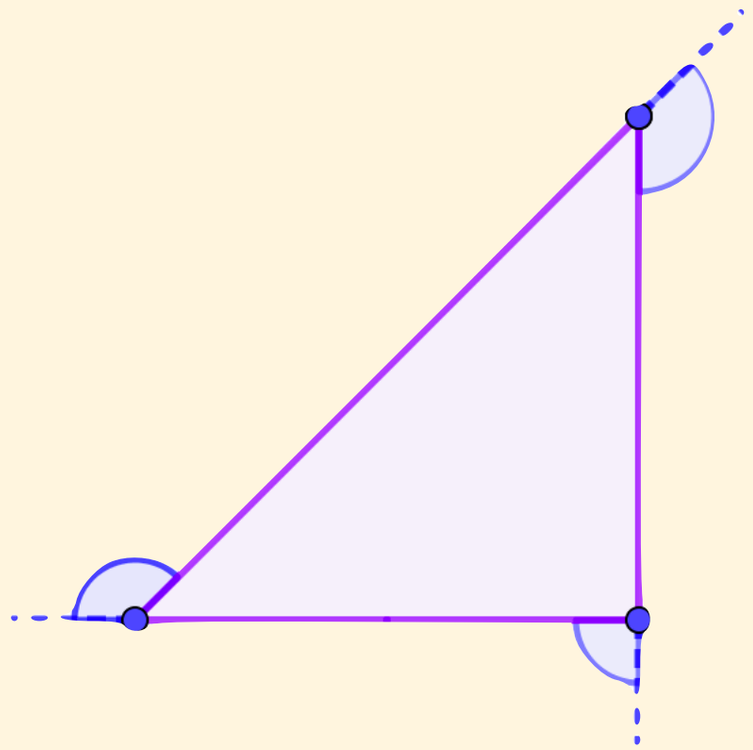
What convex tangram n-gons are possible?

triangles, quadrilaterals, pentagons, …?

Exterior Angles



$4 \times 90° = 360°$



$135° + 135° + 90° = 360°$

(turn angles)

A convex 9-gon is impossible
(proof by zombie)

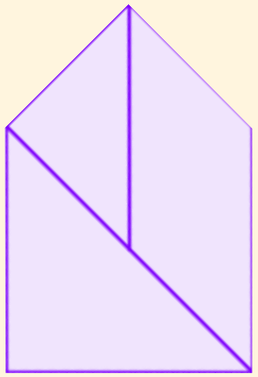
◊ All tangram angles are multiples of 45°

◊ Greatest possible interior angle: 135°

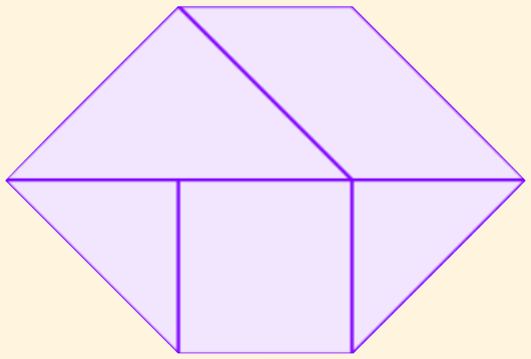
◊ Least possible exterior (turn) angle: 45°

◊ 8 x 45° = 360° so there cannot be 9 angles

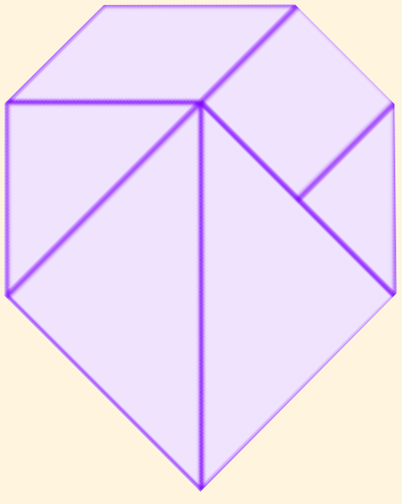
5-gon? 6-gon? 7-gon? 8-gon?



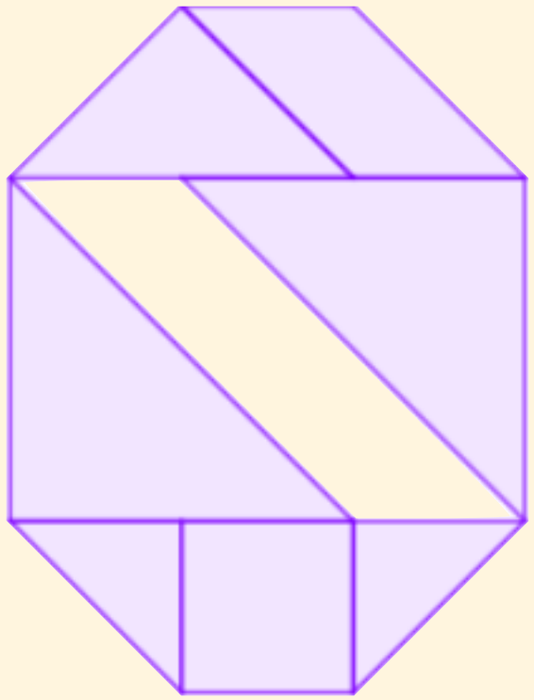
← convex 5-gon



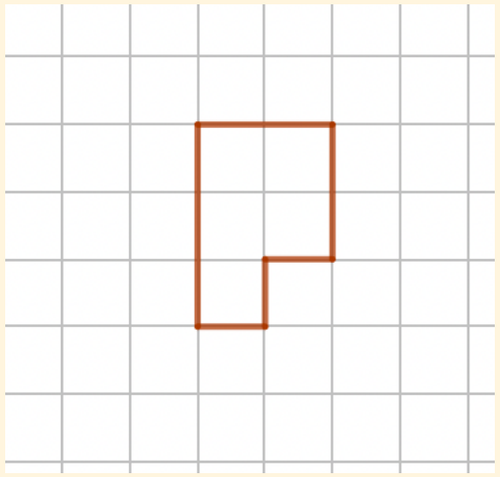
← convex 6-gon



← convex 7-gon



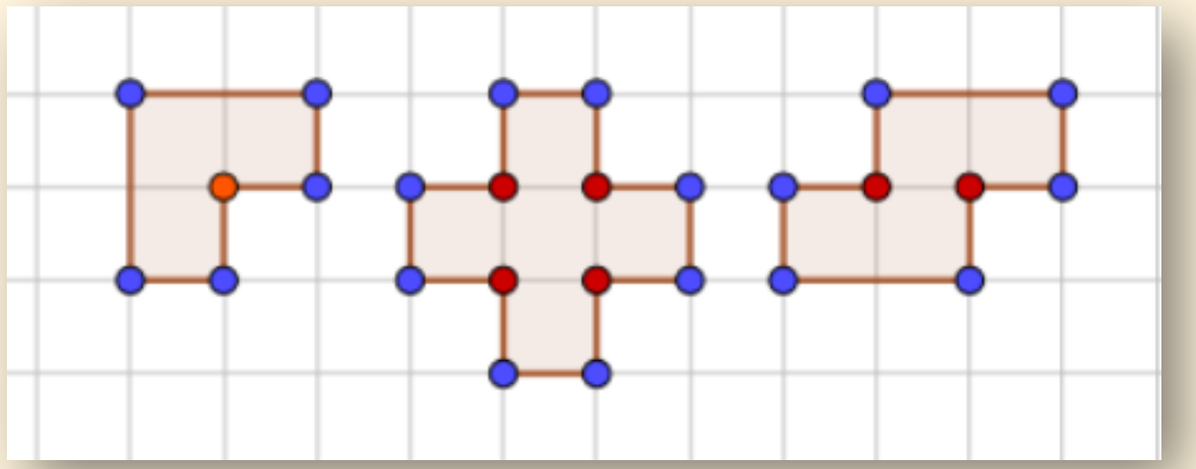
← convex 8-gon? ☹️



Polyominoes

(closed grid-paper figures:
no diagonals, no crossings)

In-corners and out-corners

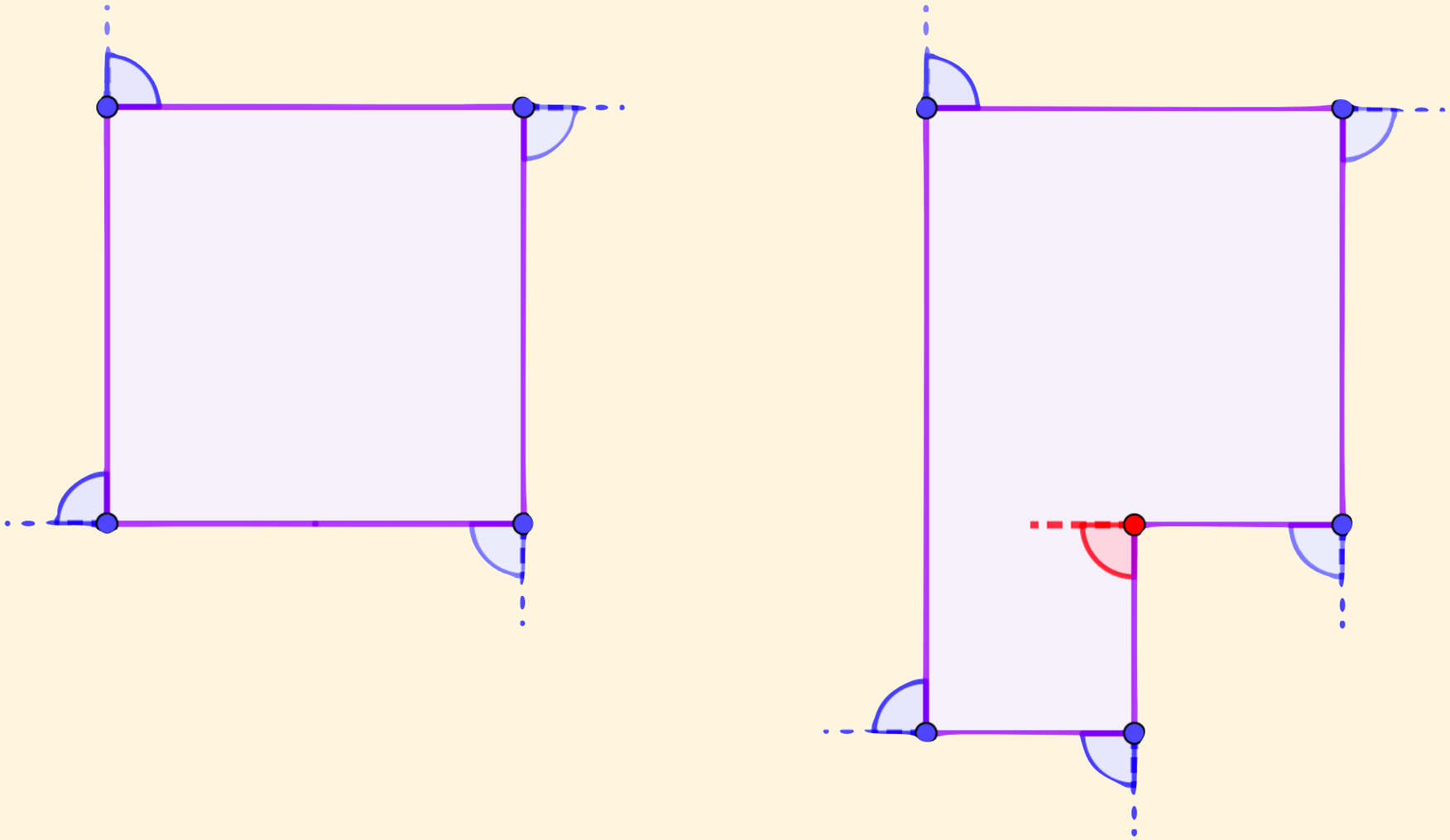


in	out
1	5
4	8
2	6

What is the pattern?

Why is it always true?

Proof by zombie again!

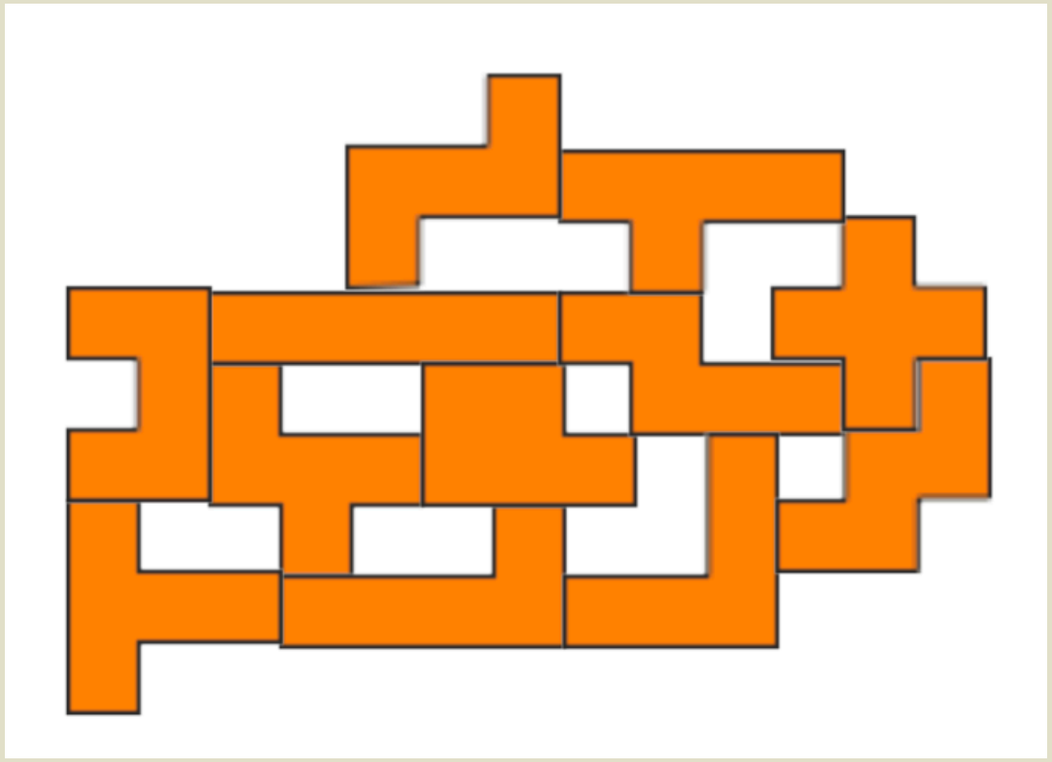


Total turning: $4 \times 90° = 360°$

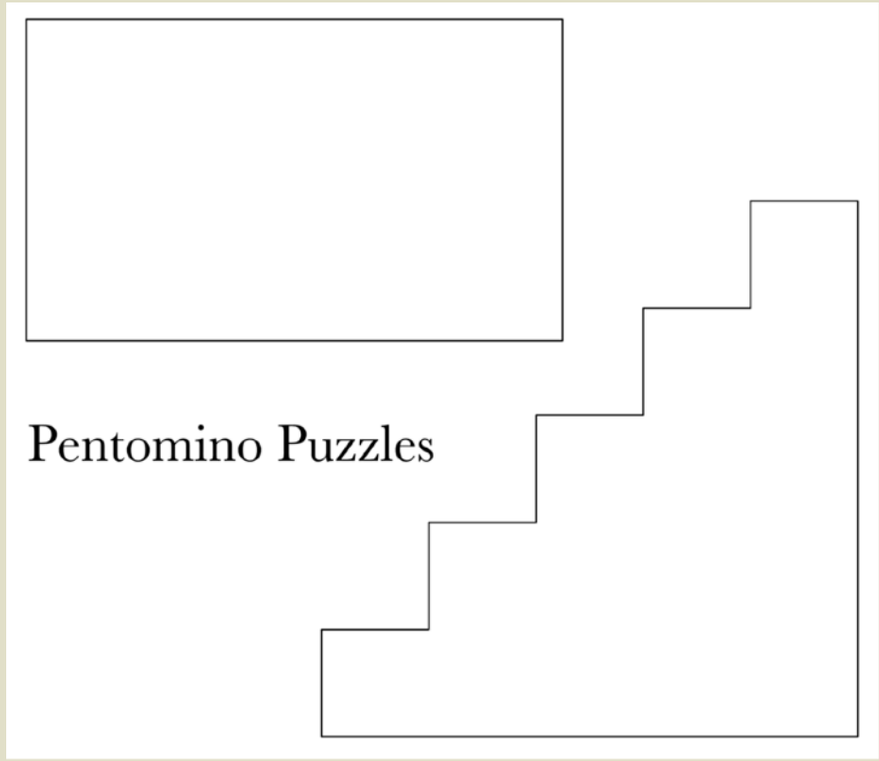
Every additional right turn must be canceled by a left turn.

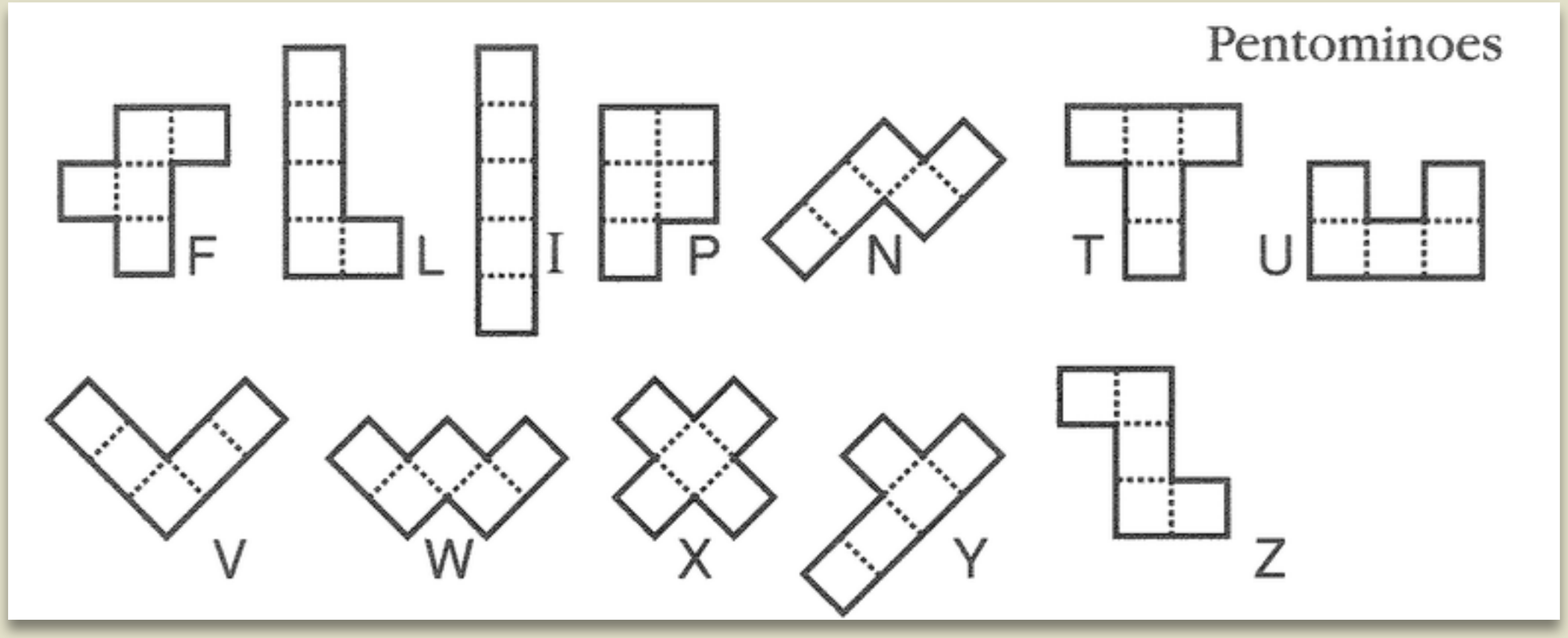
Pentominoes

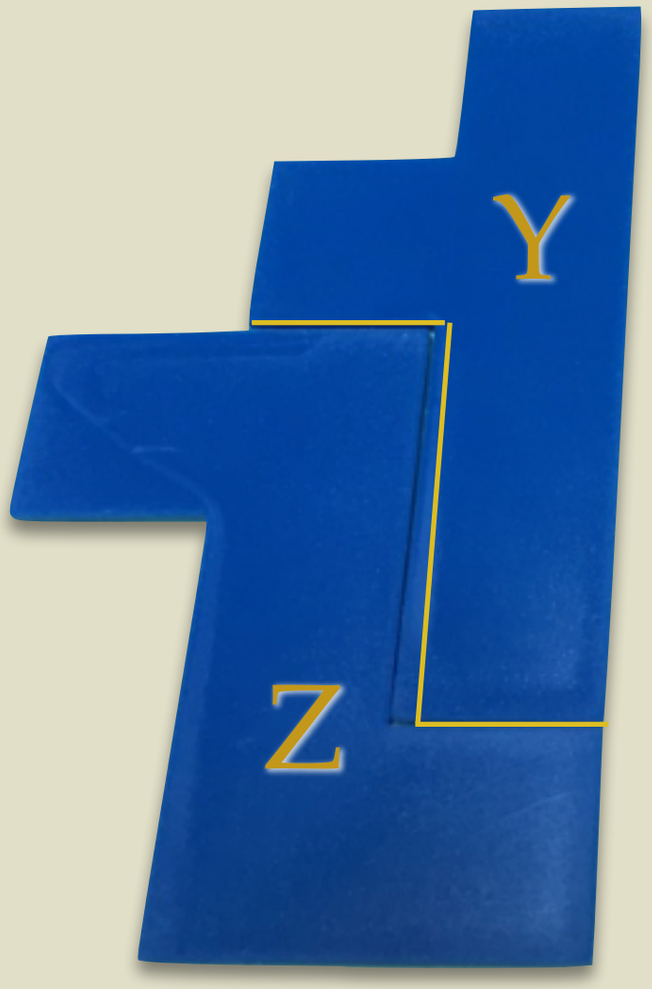
Holes



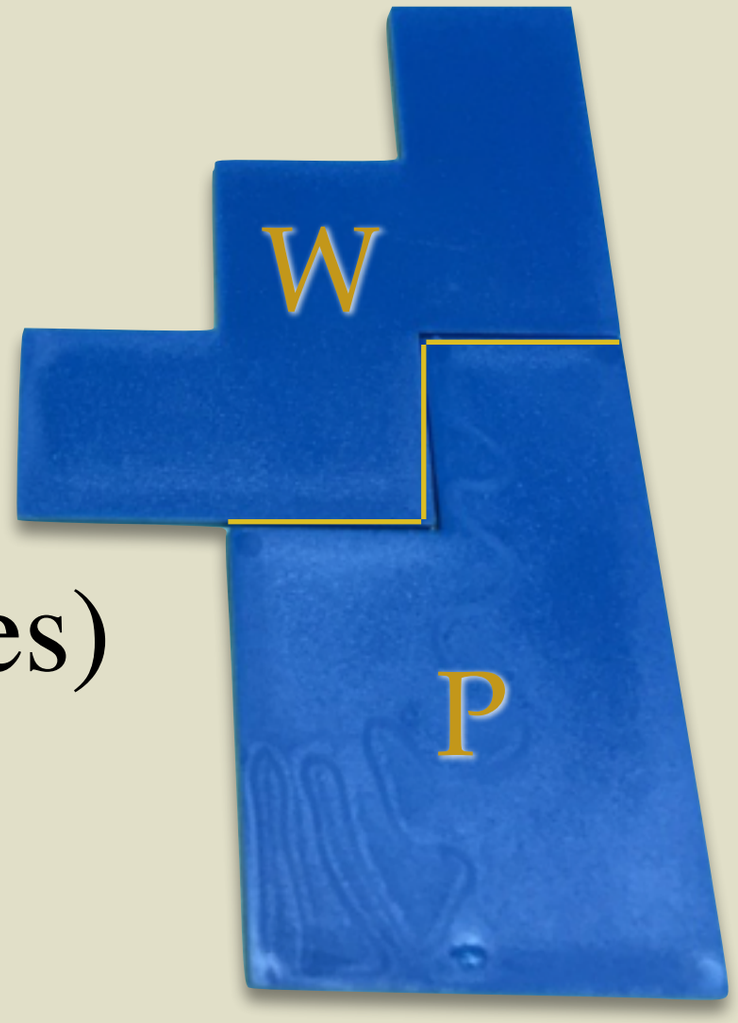
Three-piece pentomino puzzles







Layers
(congruent figures)



Generalizing

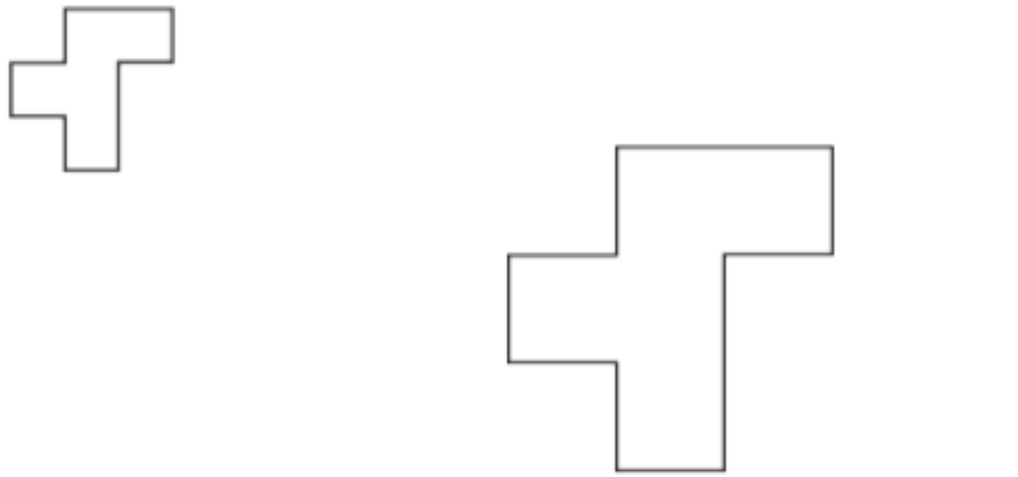
◊ What rectangles are possible?

◊ What “triangles” are possible?

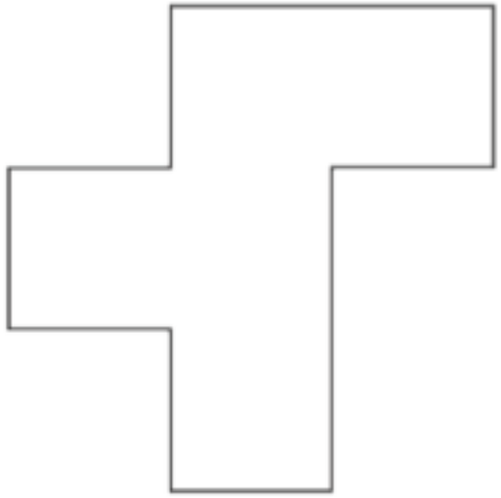
◊ What *simultaneous* rectangles?

◊ …triangles? combinations?

Pentomino Blowups

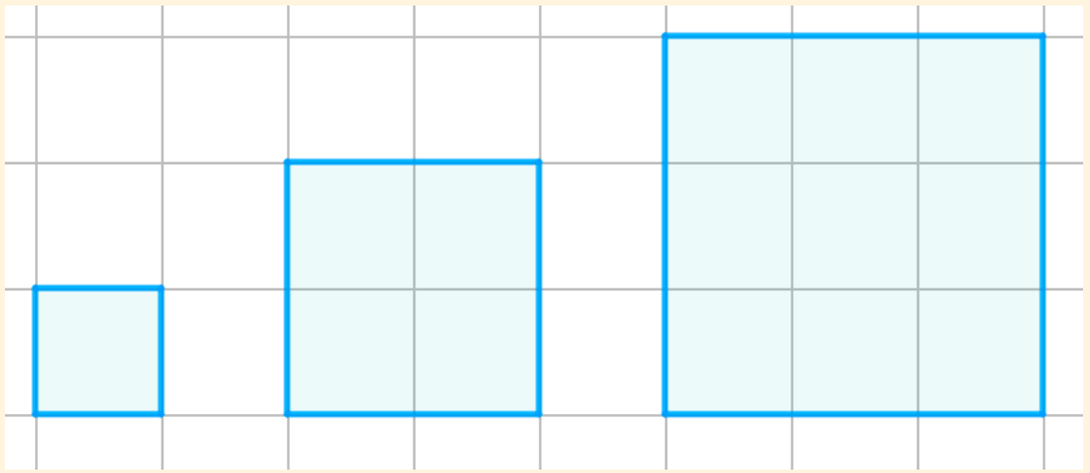


double the dimensions
(two cannot be solved)

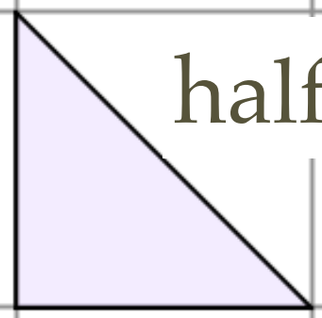


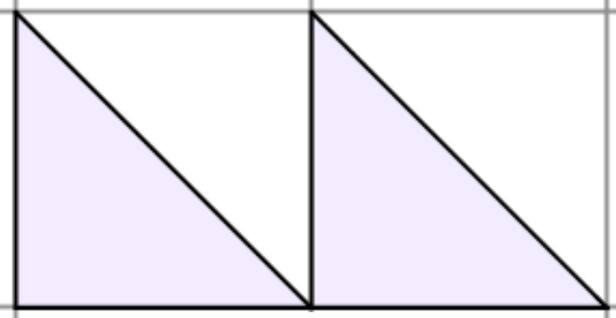
triple the dimensions

- When the dimensions are doubled, the area is multiplied by 4
- When the dimensions are tripled, the area is multiplied by 9

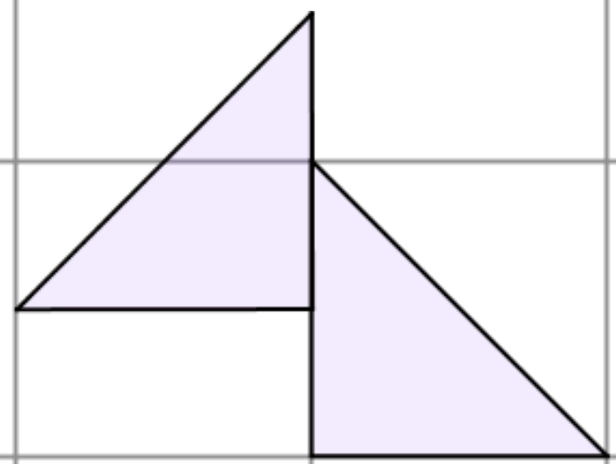


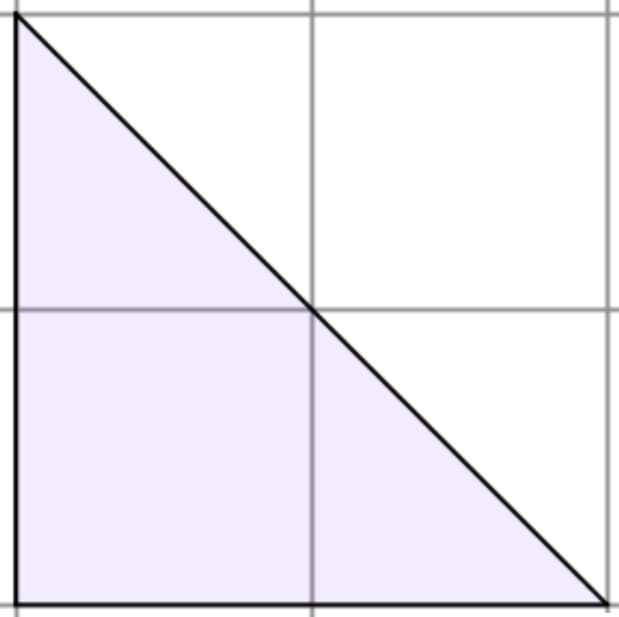
half-square





no!



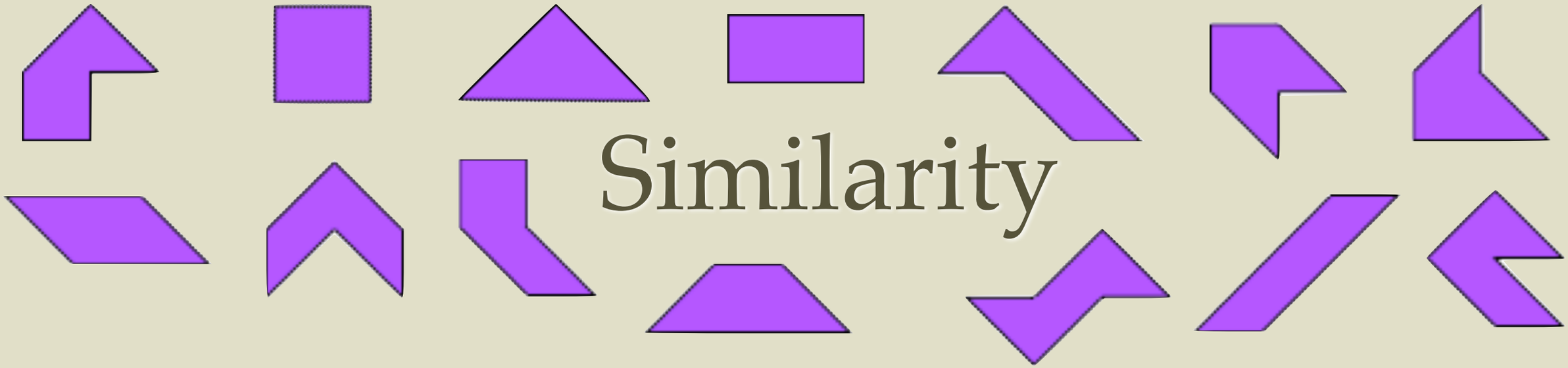


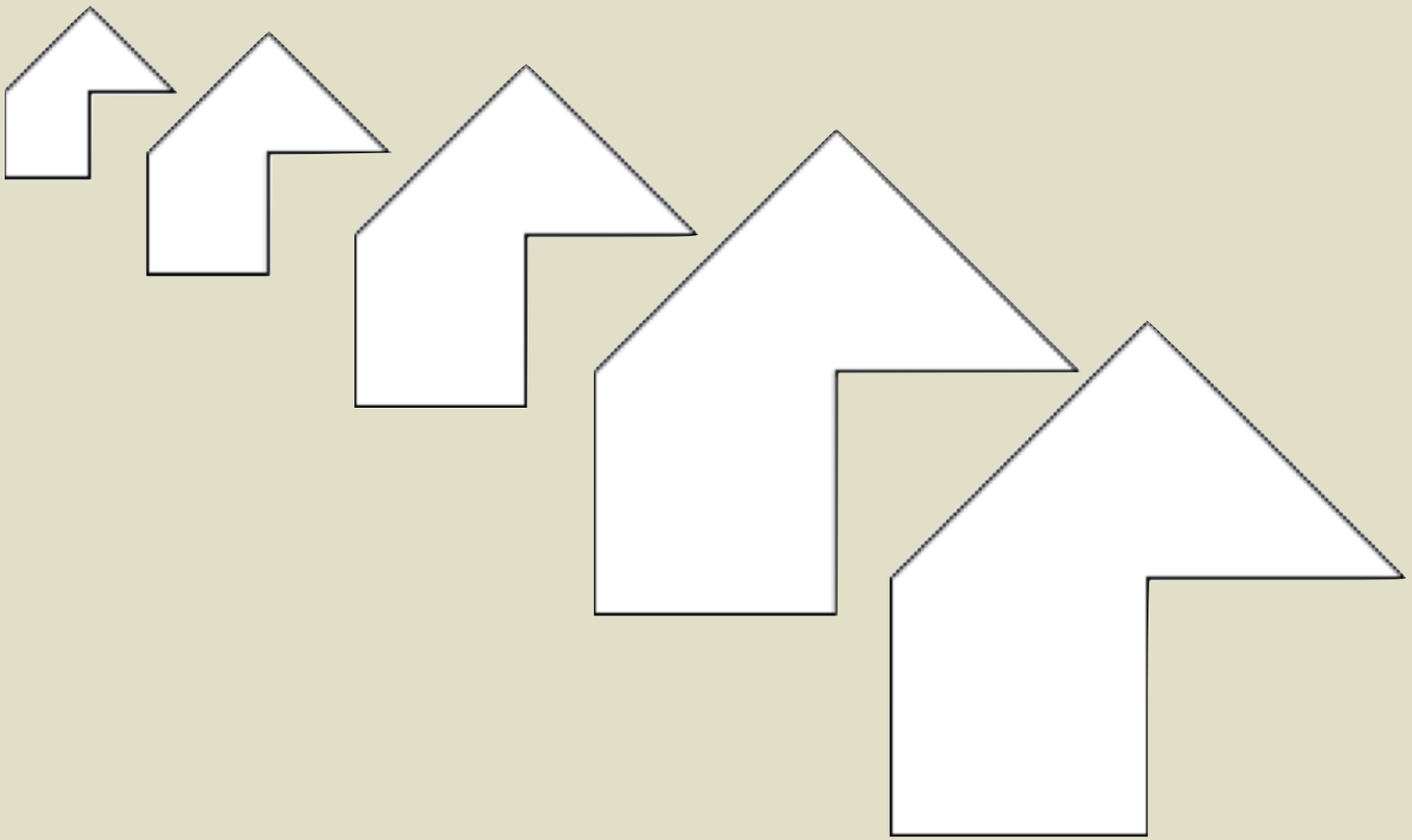
supertangram = four half-squares,

joined edge-to-edge

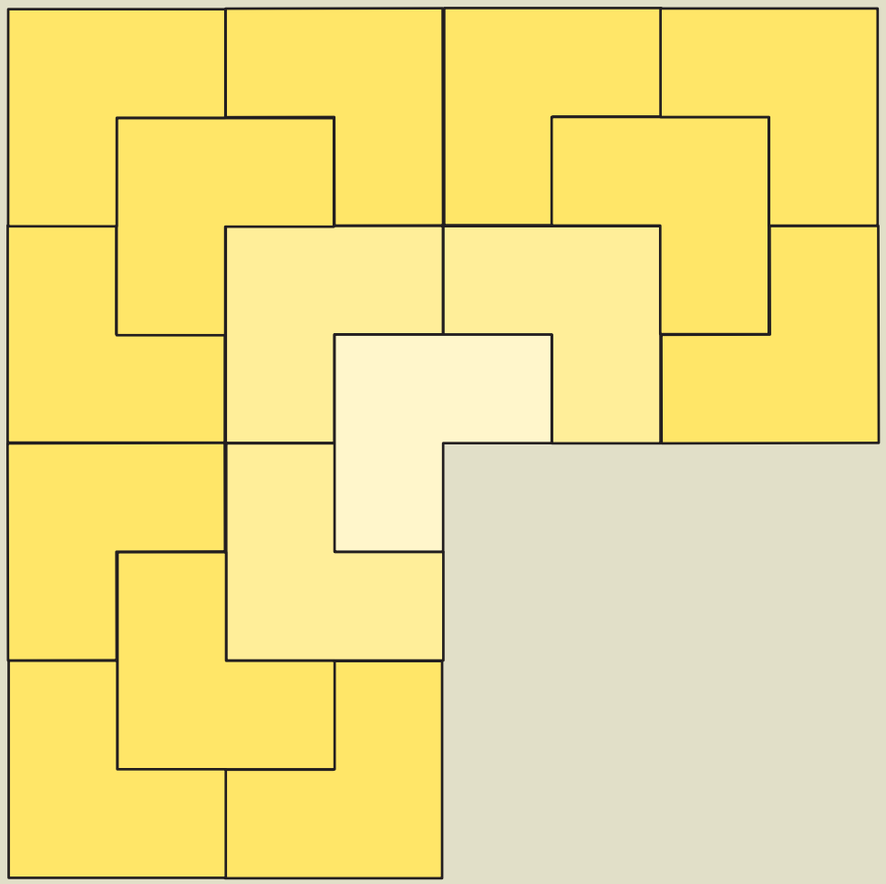
Find them all!

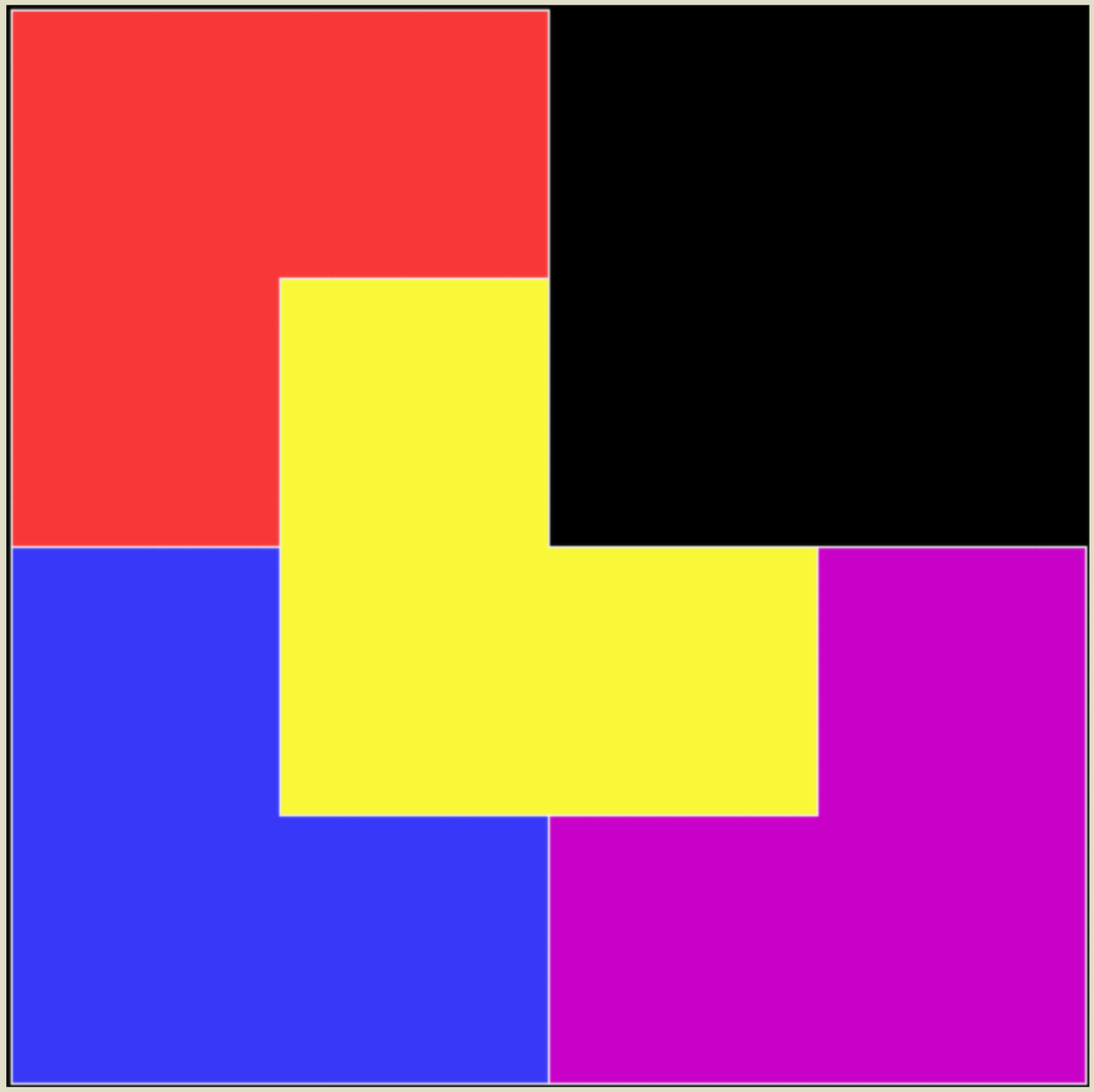
Similarity

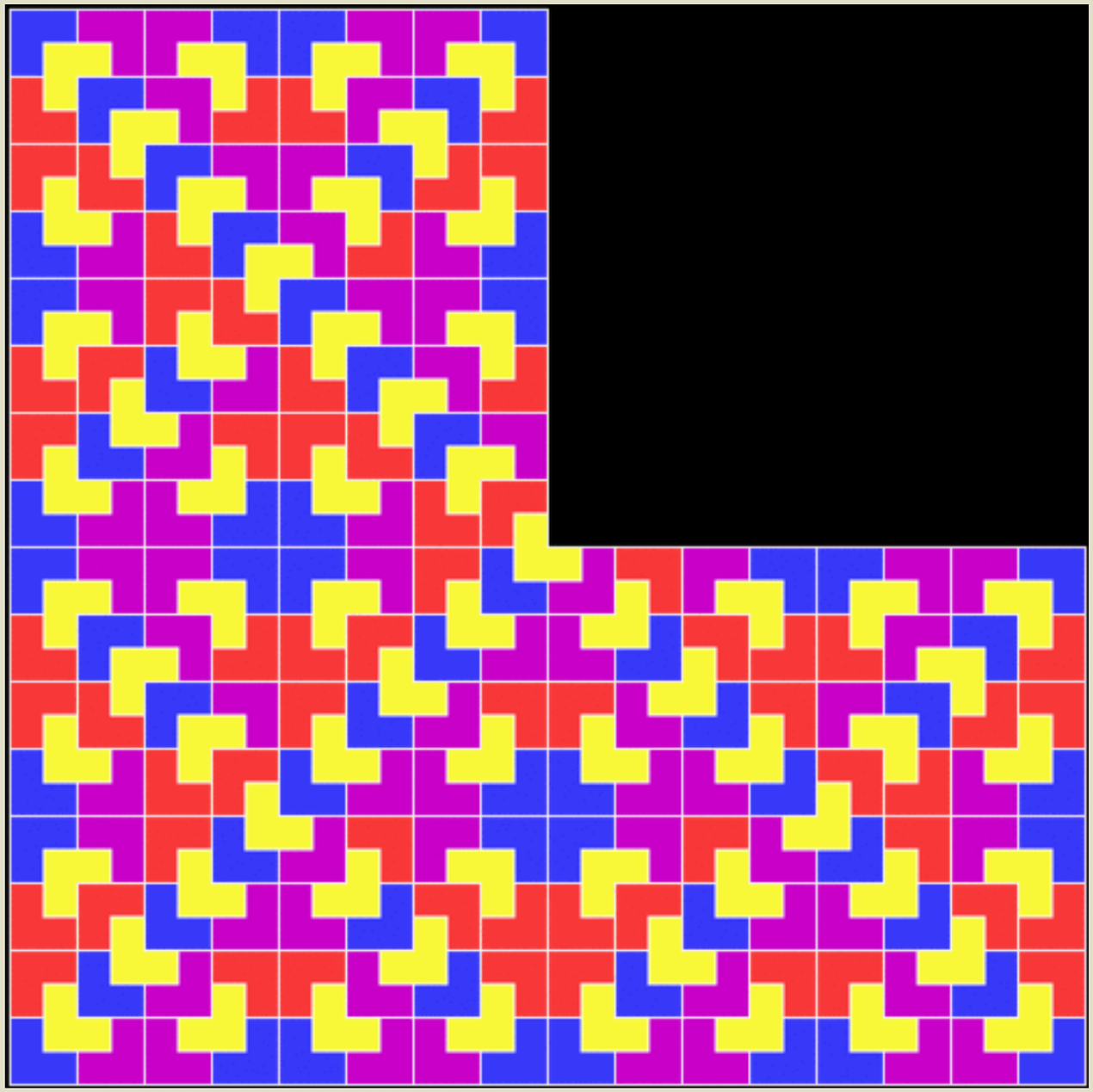




Rep-tiles

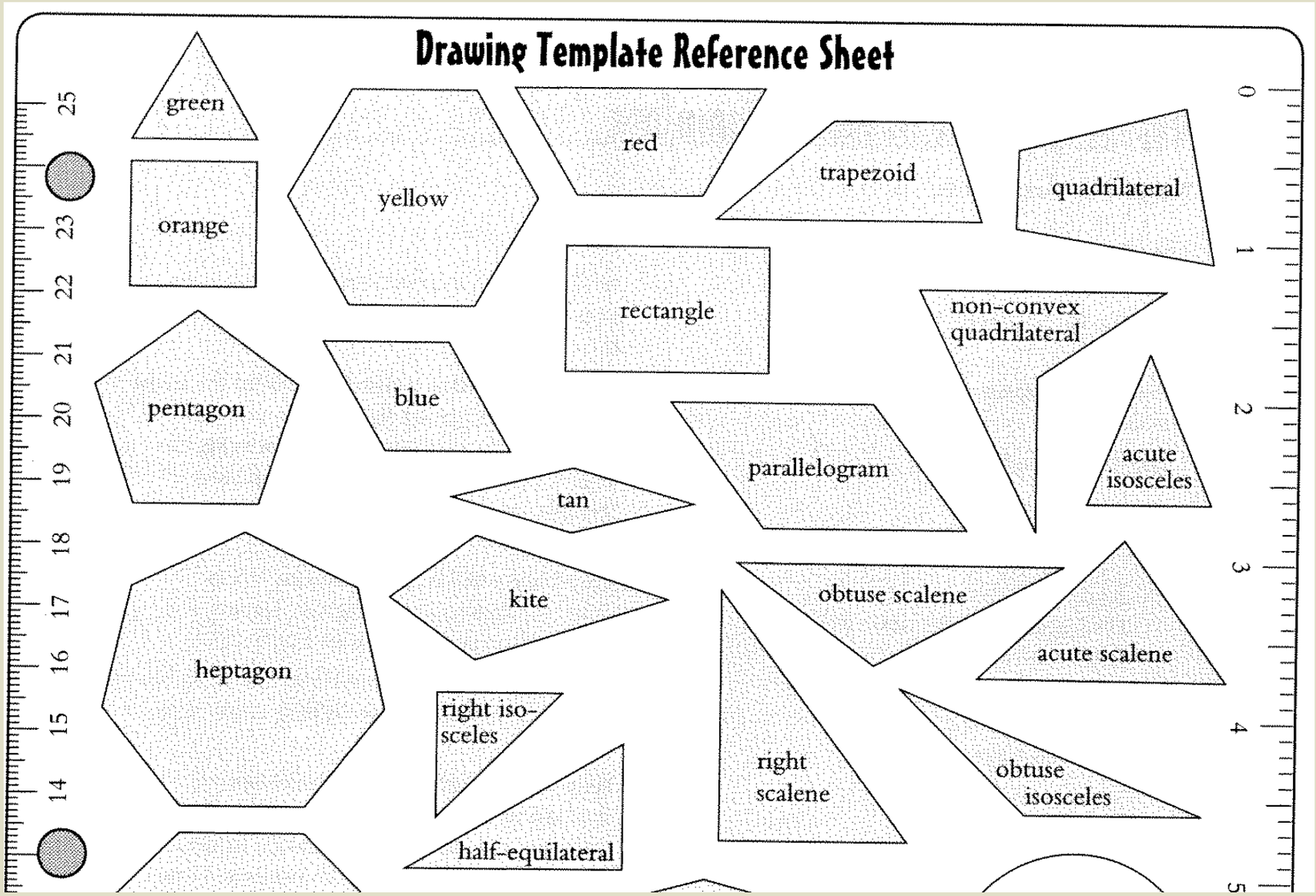






Chair tiling

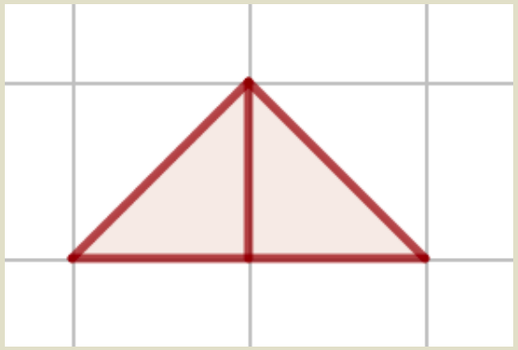
Find some rep-tiles!



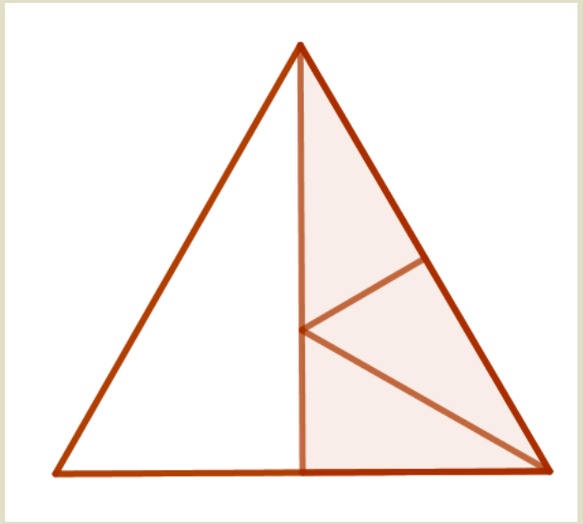
use the template, grid paper, and/or triangle paper

Rep-Triangles

Find triangles that can be tiled with
2, 3, 4, 5, …
scaled copies of themselves.



2



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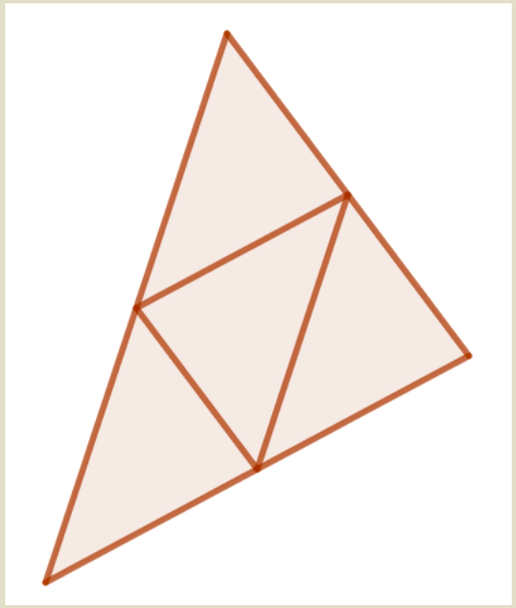

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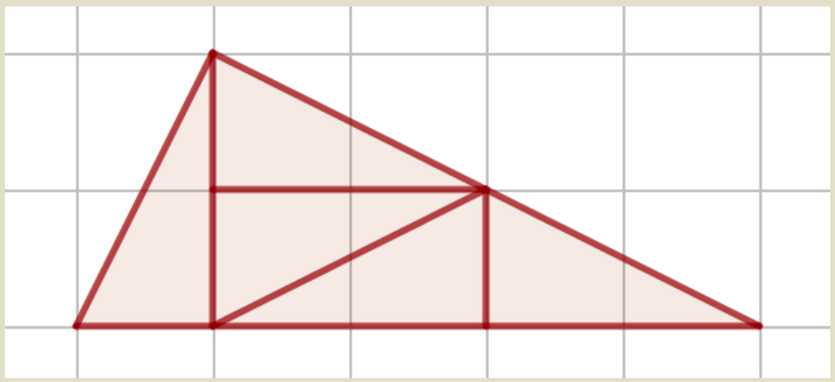
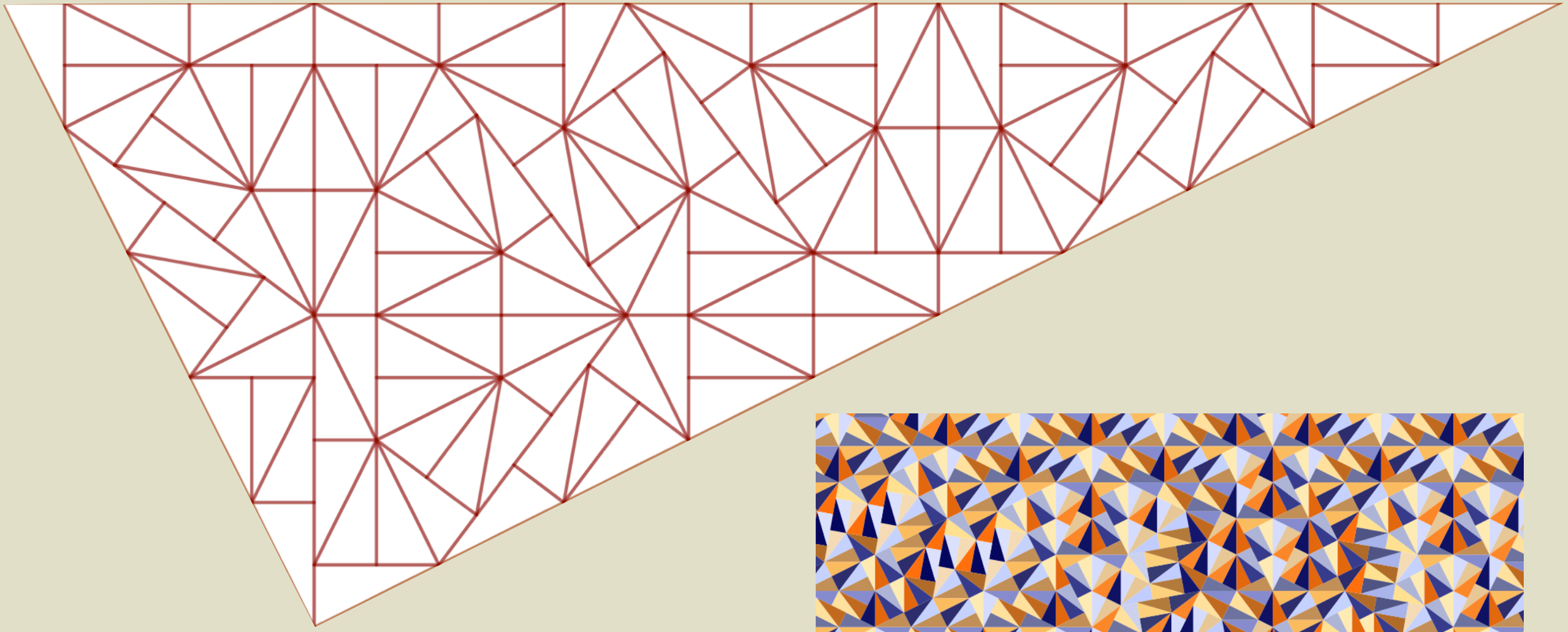
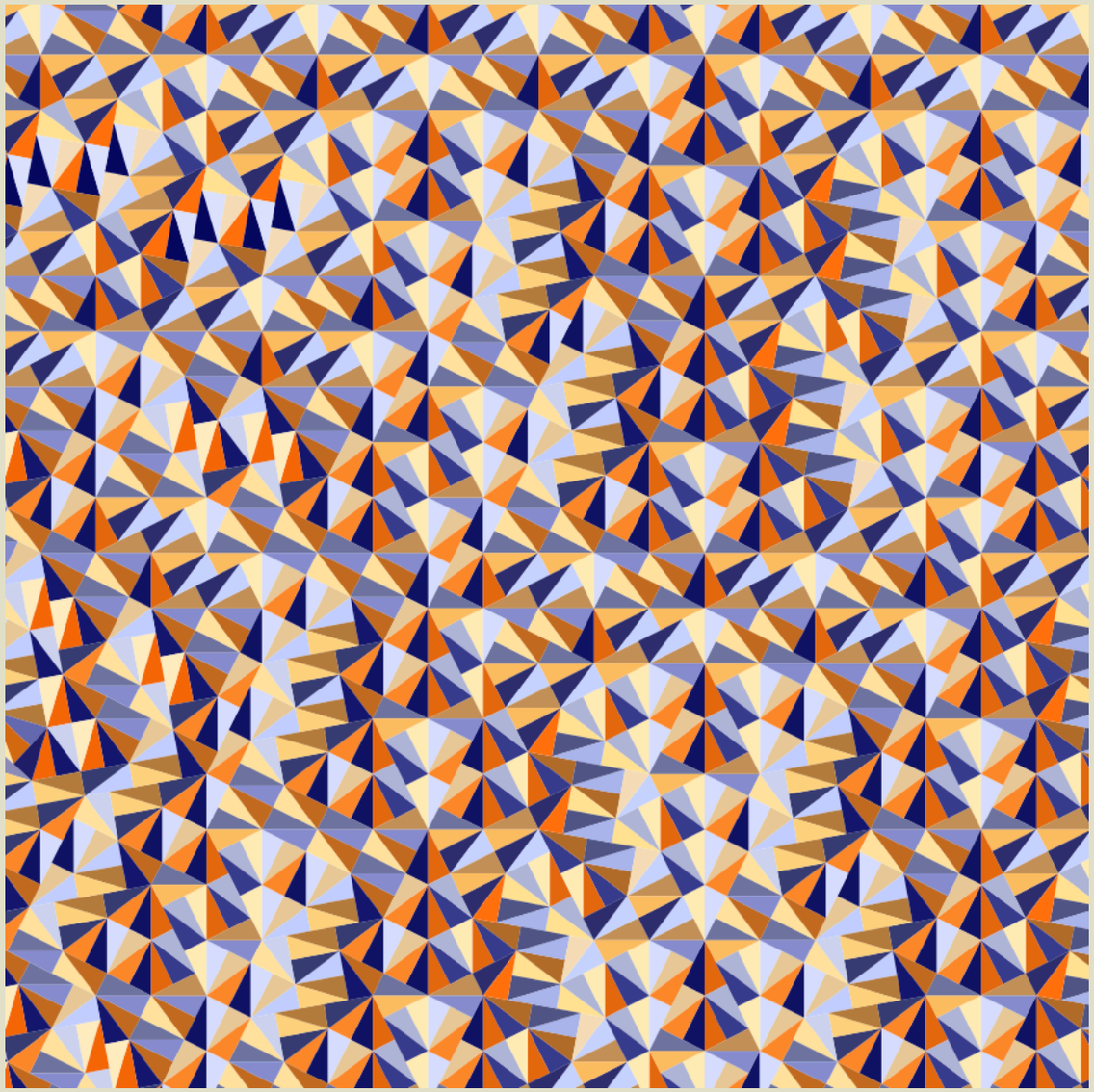


5

initial





Pinwheel tiling

Reminder:
Lots of links in the “handout”
on my **Talks** page.

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