

Geometric Puzzles

Virtually

Henri Picciotto

www.MathEducation.page

henri@MathEducation.page

blog.MathEducation.page

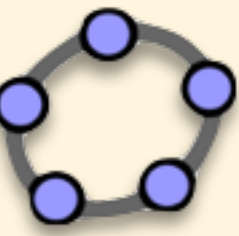
Twitter: @hpicciotto

Why Puzzles?

- ◇ Fun and interesting!
- ◇ Works with all ages
- ◇ Status equalizer
- ◇ Math content and practices

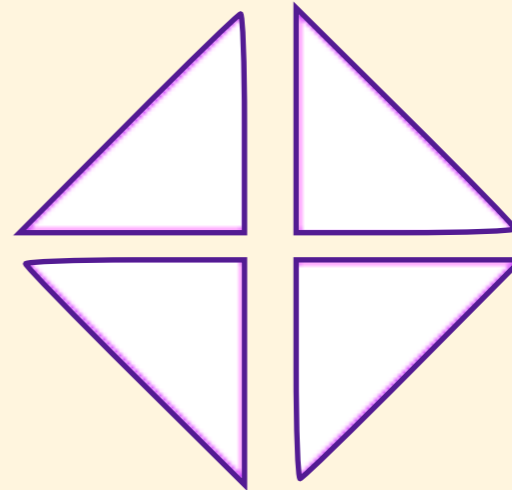
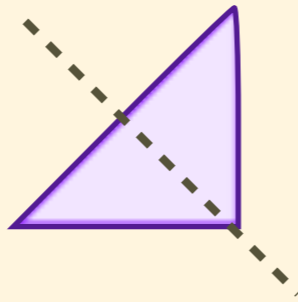
Virtual Manipulatives

vs. concrete materials

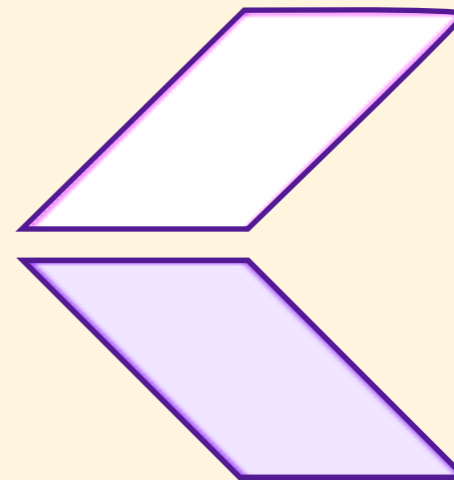
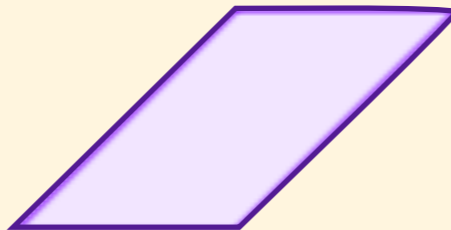



Tangrams

Line symmetric



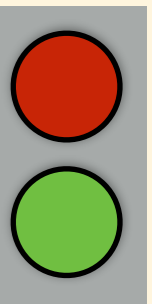
Not line symmetric



Applet 1. 

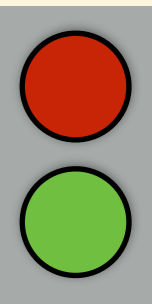
Screenshots 2. 

- ◇ Make a geometric figure, using 1 to 7 pieces
- ◇ Take a screenshot
- ◇ Share on the appropriate page, indicating the number of pieces
- ◇ Add pages if needed



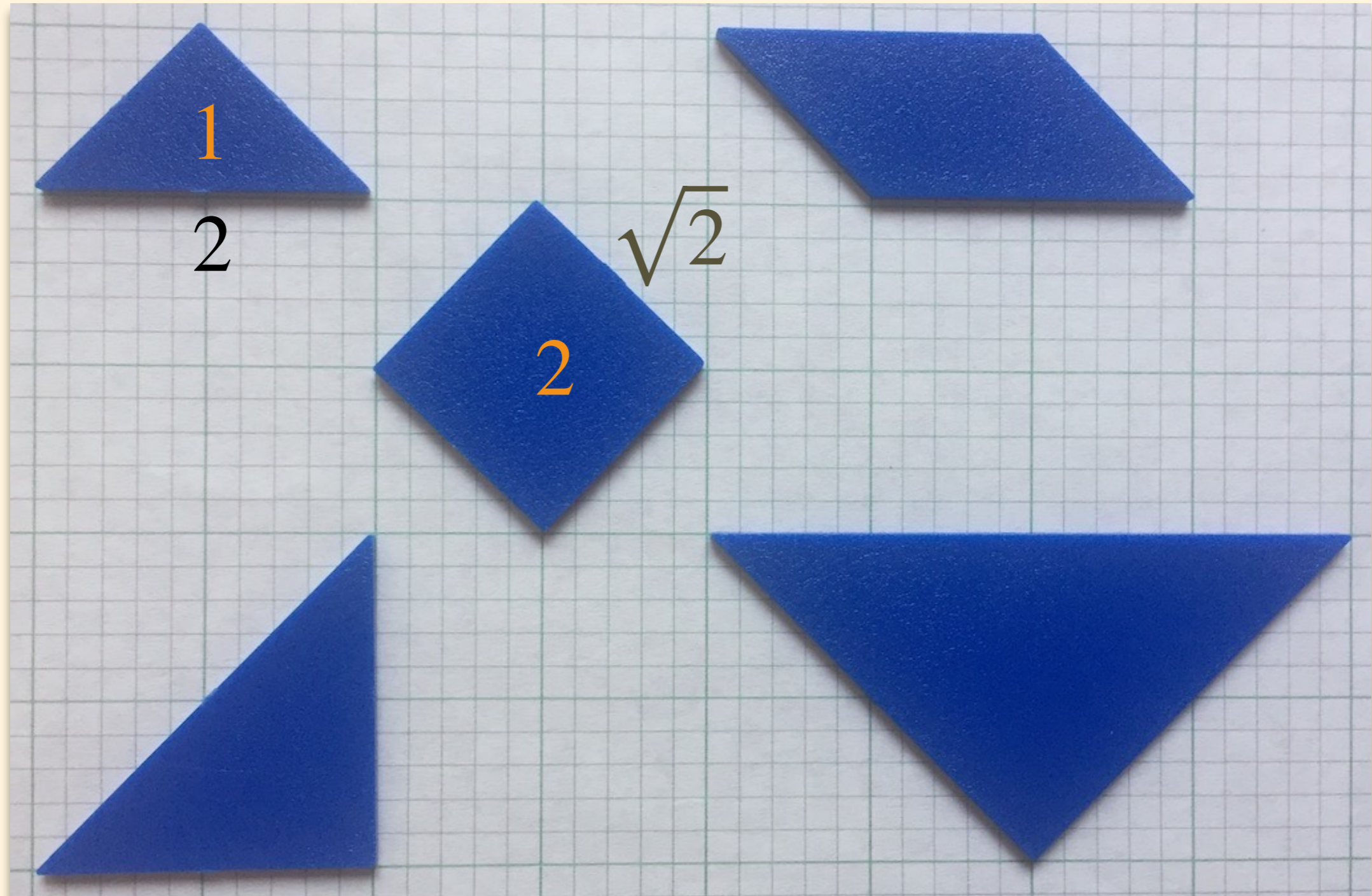


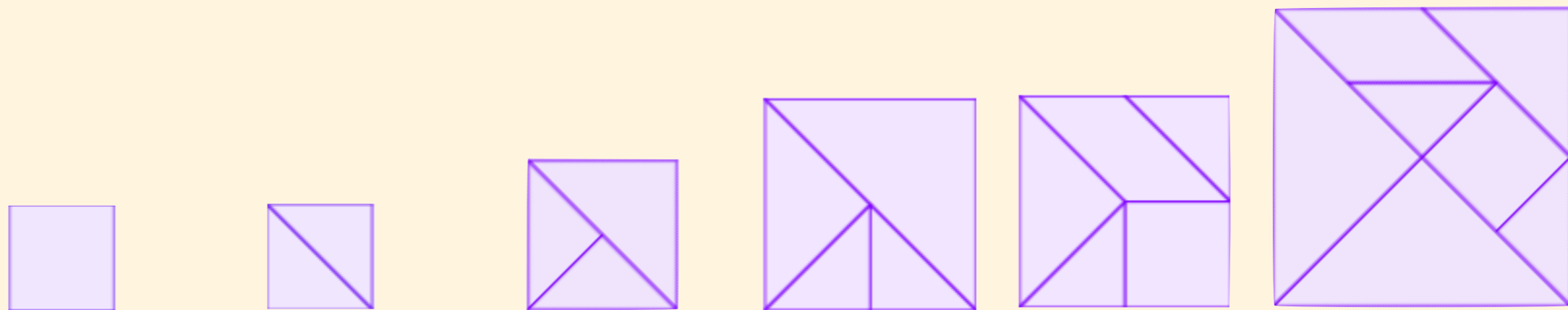
What squares are possible?



Tangram Measurements

(inches and square inches)





Pieces	1	2	3	4	5	7
Area	2	2	4	8	8	16
Side	$\sqrt{2}$	$\sqrt{2}$	2	$2\sqrt{2}$	$2\sqrt{2}$	4

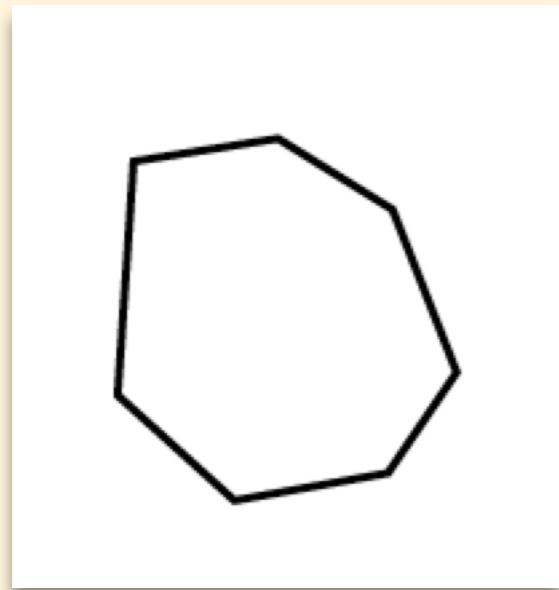
$$\sqrt{8} = 2\sqrt{2}$$

A 6-piece square is impossible

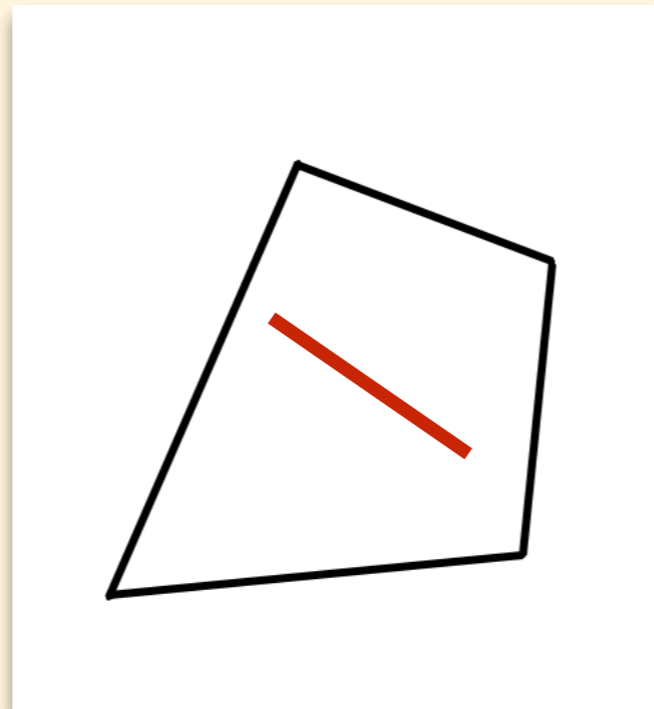
- ◇ Total tangram area: 16 in^2
- ◇ 6-piece area: 15, 14, or 12 in^2
- ◇ Cannot be a tangram square!

Convex Polygons

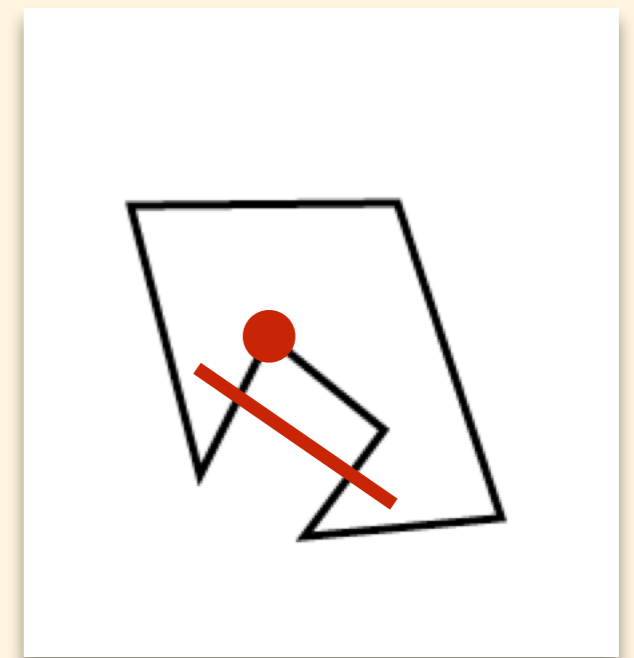
All angles $< 180^\circ$



yes

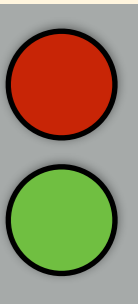


yes

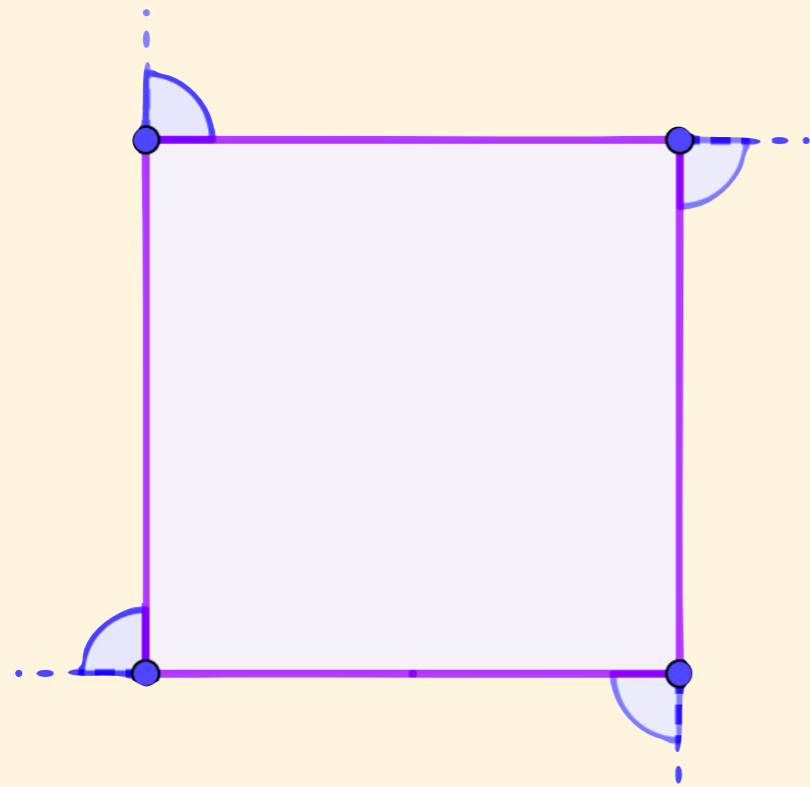


no

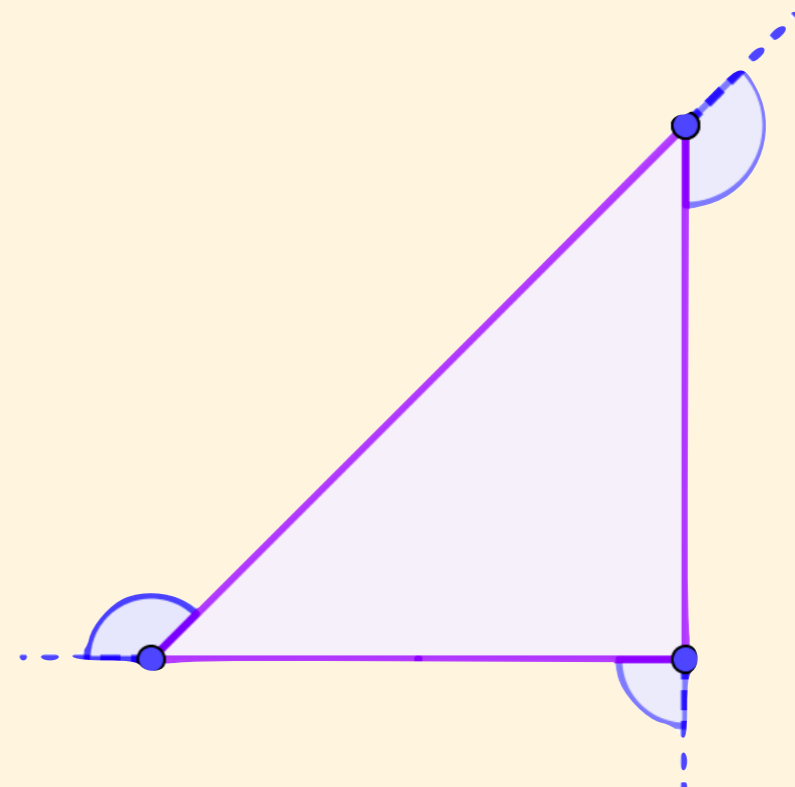
What convex tangram
figures are possible?
triangles, quadrilaterals, pentagons, ...?



Exterior Angles



$$4 \times 90^\circ = 360^\circ$$



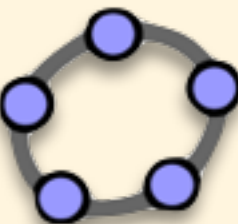
$$135^\circ + 135^\circ + 90^\circ = 360^\circ$$

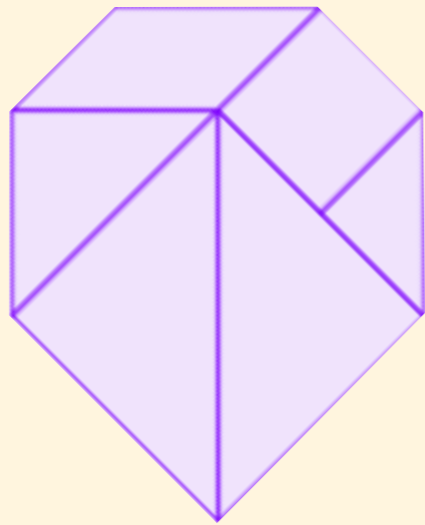
(turn angles)

A convex 9-gon is impossible (proof by zombie)

- ◇ All tangram angles are multiples of 45°
- ◇ Greatest possible interior angle: 135°
- ◇ Least possible exterior (turn) angle: 45°
- ◇ $8 \times 45^\circ = 360^\circ$ so there cannot be 9 angles

A convex 8-gon is impossible (proof by trying to do it!)



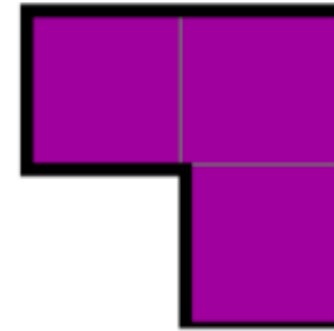


← convex 7-gon

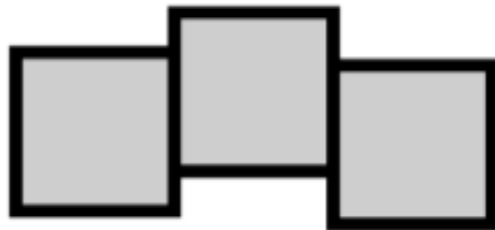
This is a domino



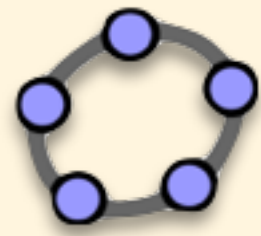
These are triominoes



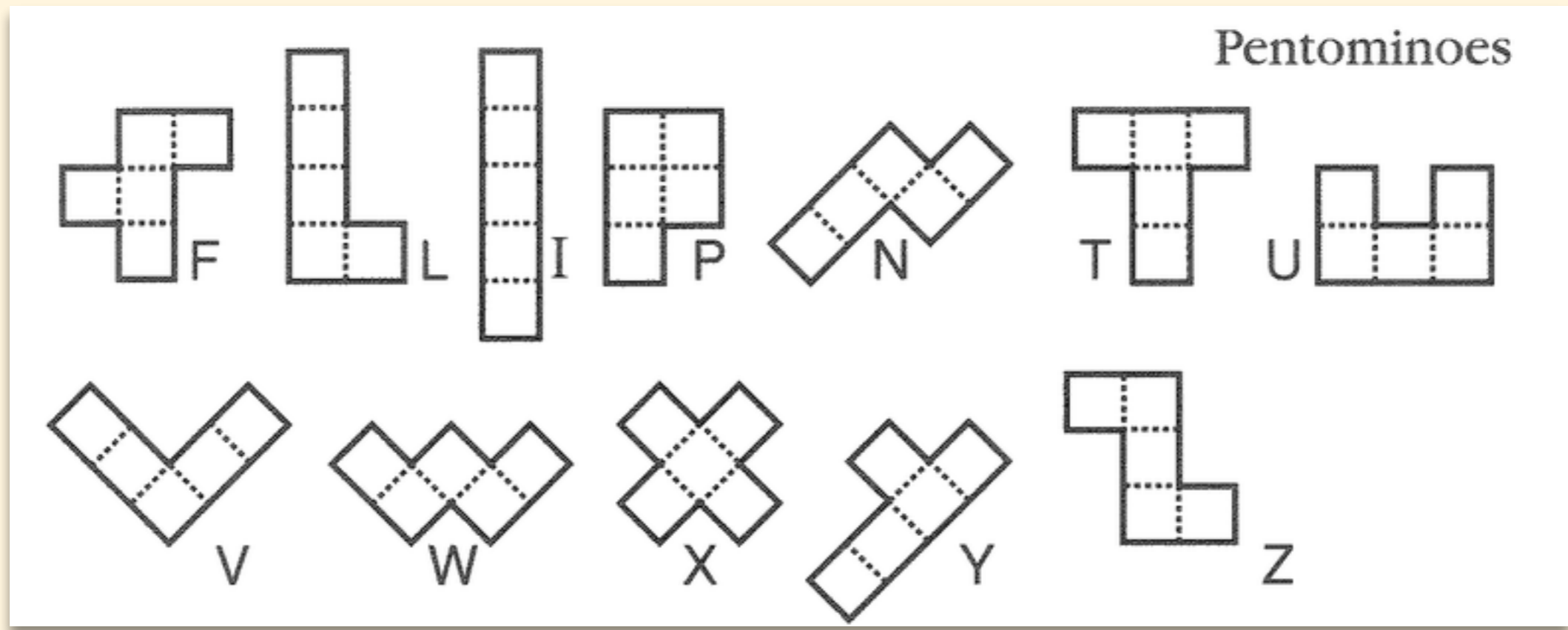
These are *not* polyominoes



Polyominoes are shapes that are made by joining squares edge-to-edge.



Pentominoes



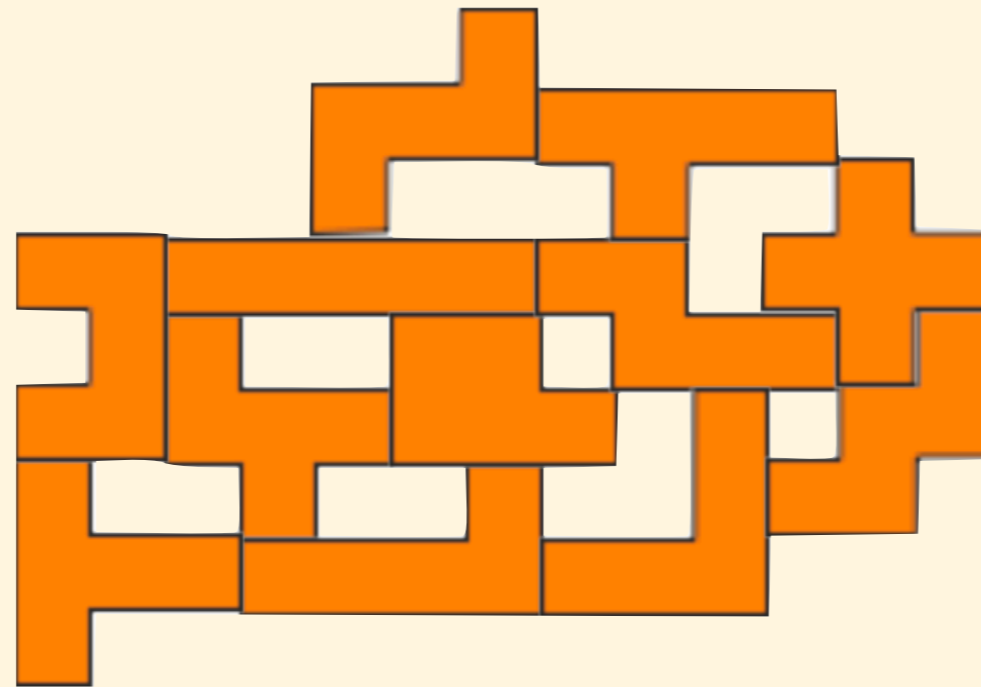
Applet 3.



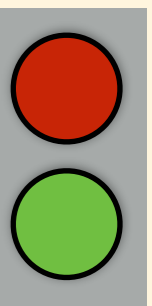
Screenshots 4.



Holes

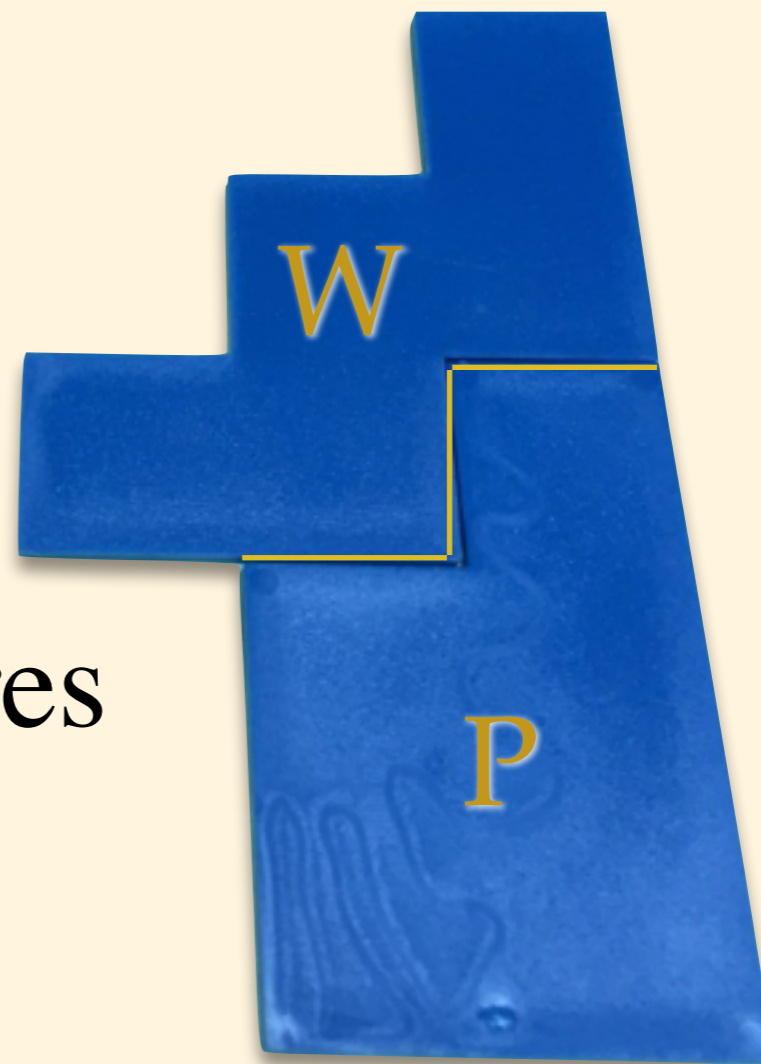


8 holes. Can you get more?

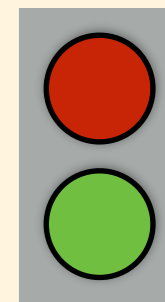




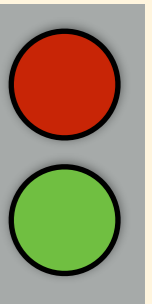
Congruent Figures



Triples?



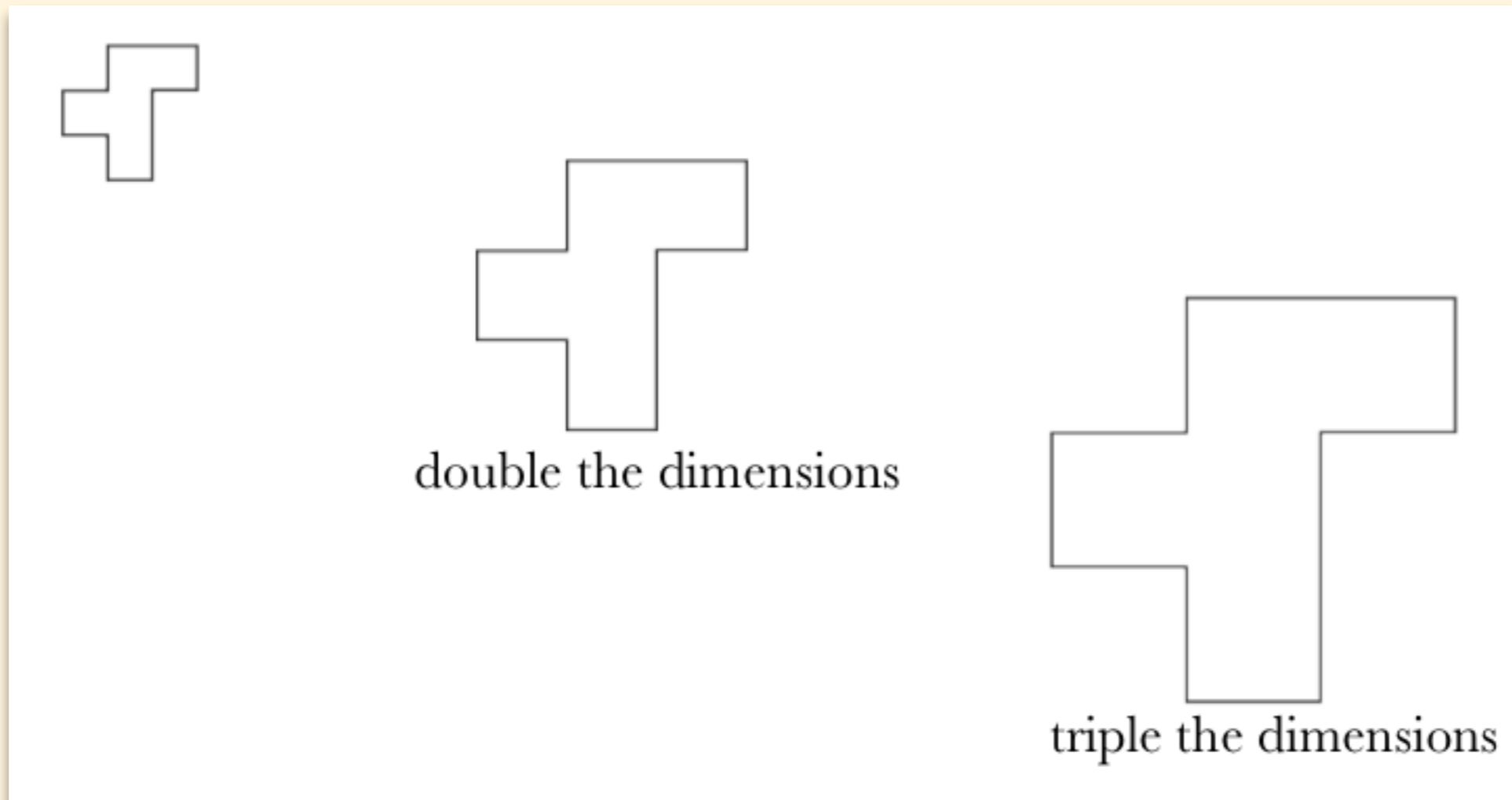
Pentomino Rectangles



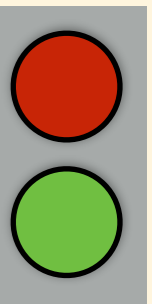
Pentomino Rectangles

- ◇ If a figure can be covered with pentominoes, what can you say about its area?
- ◇ If a figure can be covered with pentominoes, what can you say about its sides?
- ◇ If a rectangle can be covered with pentominoes, what can you say about its sides?

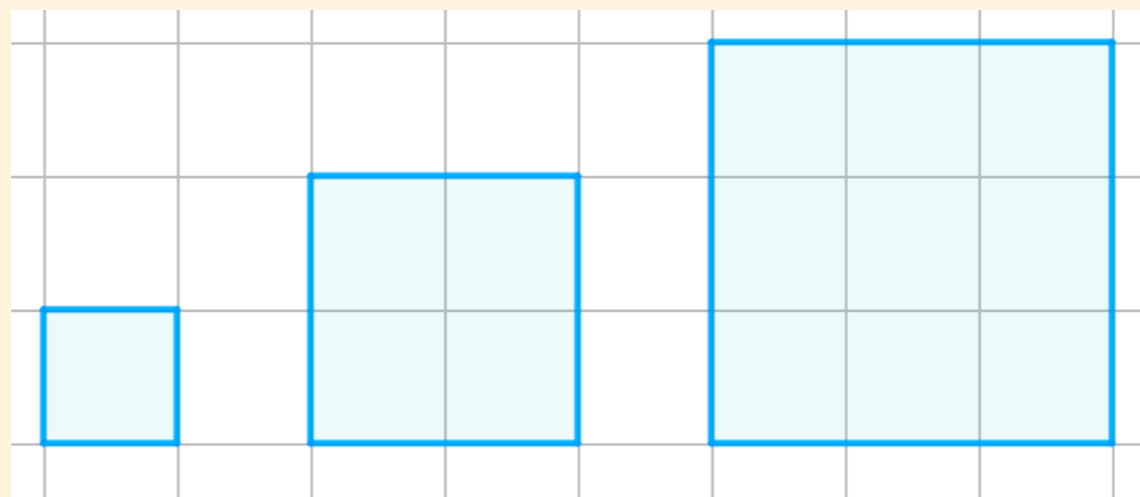
Pentomino Blowups

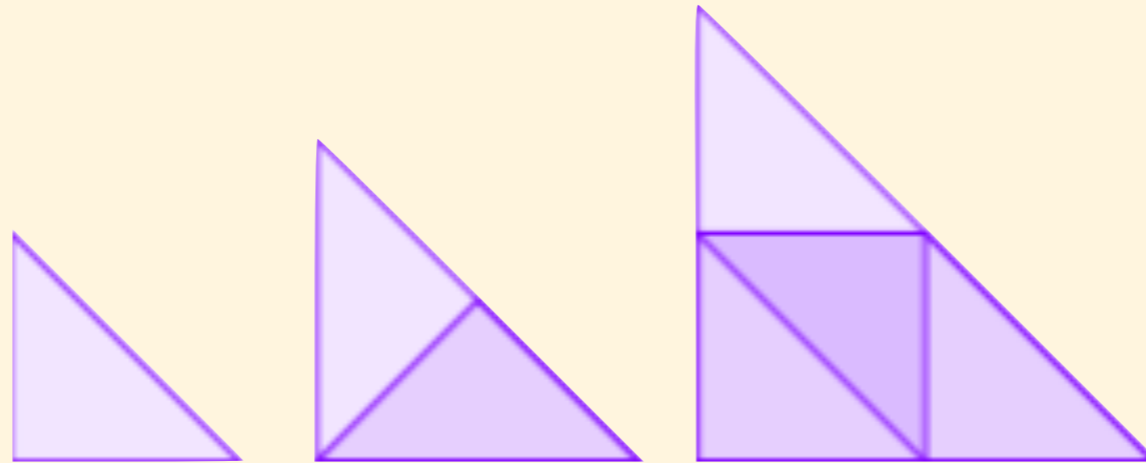


Double the dimensions (two cannot be solved)
Triple the dimensions (all are possible, but hard!)



- ◇ When the dimensions are doubled, the area is multiplied by 4
- ◇ When the dimensions are tripled, the area is multiplied by 9

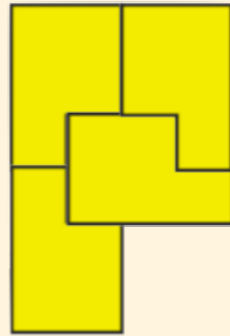




◇ Ratio of areas?

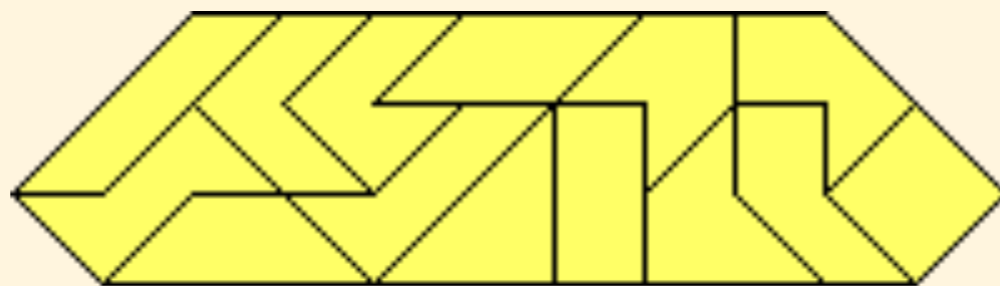
◇ Scaling factor?

Taking it further!



◇ Rep-tiles

◇ Supertangrams



Lots of links in the “handout”
on my **Talks** page.

Henri Picciotto

www.MathEducation.page

henri@MathEducation.page

blog.MathEducation.page

Twitter: @hpicciotto

