Three Quadratic Forms

Only use an electronic grapher to check your answers!

Factored Form

1. For the function $y = 2(x + 3)(x - 4)$:
   a. What are the *roots* (the x-intercepts)? (Hint: what is the value of y at the x-intercepts?)
   b. What is the y-intercept?
   c. What is the x-coordinate of the vertex? (Hint: it's half way between the roots)
   d. What is the y-coordinate of the vertex?

The following questions are about *factored form*: $y=a(x - p)(x - q)$

2. How do we know p and q are the roots?

3. Find, in terms of a, p, and q:
   a. The y-intercept.
   b. The x-coordinate of the vertex.
   c. **Challenge**: The y-coordinate of the vertex.

From Factored Form to Standard Form

$y = ax^2 + bx + c$ is called *standard form*. Some of the information that is obvious in factored form is not as visible in standard form.

4. Take the equation $y = a(x - p)(x - q)$, and distribute, so as to write it in standard form. [Hint: this is a two-step process. One way to do it is to first multiply $a(x-p)$. Then multiply the product by $(x - q)$.]

5. Write formulas for b and c in terms of a, p, and q. The trickiest part is to figure out b — it requires factoring the x.

6. Find, in terms of a, b, and c:
   a. The y-intercept.
   b. The sum of the roots.
   c. The product of the roots.
   d. The x-coordinate of the vertex.
   e. **Challenge**: The y-coordinate of the vertex.

Note that the x-coordinate of the vertex does not depend on c. It follows that the formula still works even if the function has no roots: by changing c, we can change it to a function that does have roots without affecting the x-coordinate of the vertex.

Vertex Form

7. Consider the expression $2(x-3)^2+4$
   a. What is the smallest value it can possibly be? Explain, without referring to a graph.
   b. For what value of x does it reach this smallest value? Explain, without referring to a graph.

8. Consider the expression $a(x-h)^2 + v$, with $a > 0$
   a. What is the smallest value it can possibly be? Explain.
   b. For what value of x does it reach this smallest value? Explain.
   c. What happens if $a < 0$?

It follows that (h, v) are the coordinates of the vertex of the parabola with equation $y = a(x-h)^2 + v$. This is *vertex form*.

From Vertex Form to Standard Form

Some of the information that is obvious in vertex form is not as visible in standard form.

9. Take the equation $y = a(x-h)^2 + v$, and distribute, so as to write it in standard form.

10. Write formulas for b and c in terms of a, h, and v.

11. Find h in terms of a and b.

12. **Challenge**: Find v in terms of a, b, and c.

Are There Roots?

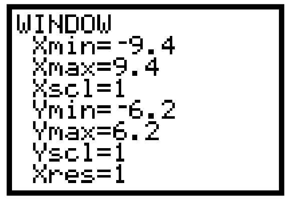
The y-coordinate of the vertex, which you might have found in #6e or 12 is $v = \dfrac{-b^2 + 4ac}{4a}$.
If you got a different answer, figure out whether you made a mistake, or whether your solution was equivalent to this one.

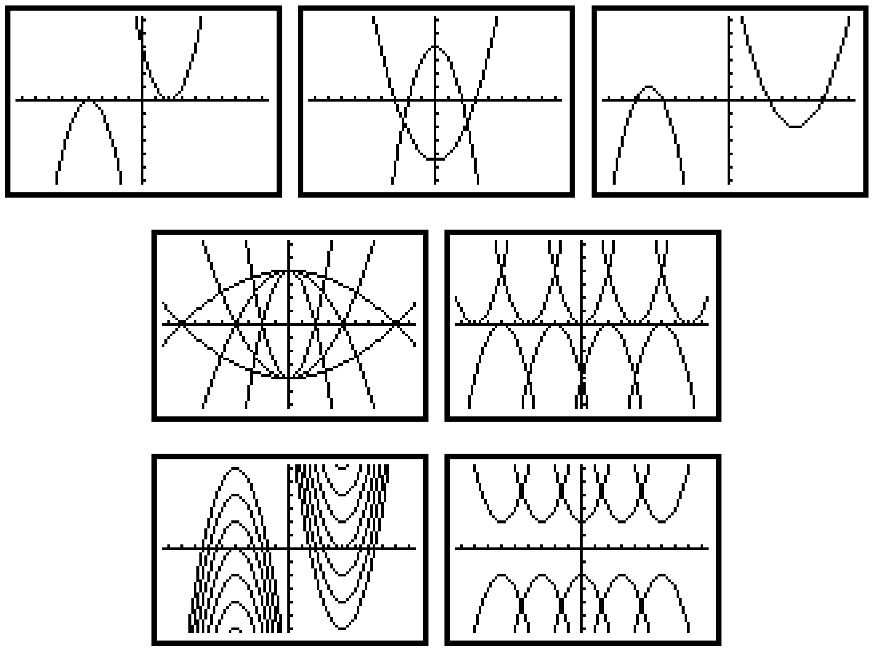
We will use a and v to determine if a quadratic function has roots.

13. Since $a \neq 0$, there are two cases:
   a. If $a > 0$, we have a "smile" parabola. In this case, the parabola intersects the x-axis if _______
   b. If $a < 0$, we have a "frown" parabola. In this case, the parabola intersects the x-axis if ________

14. **Challenge**: Find an inequality involving only a, b, and c that one can use to figure out whether the parabola intersects the x-axis. (Hint: It can be deduced from the two cases above and the formula for v.)

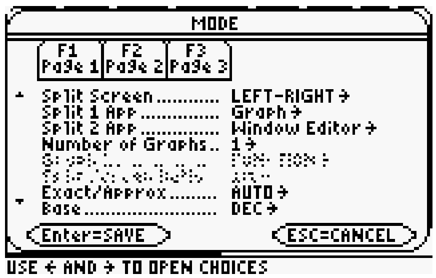
Make These Parabolas (TI-83/84)





Make These Parabolas (TI-89)

To split the screen: MODE F2 then



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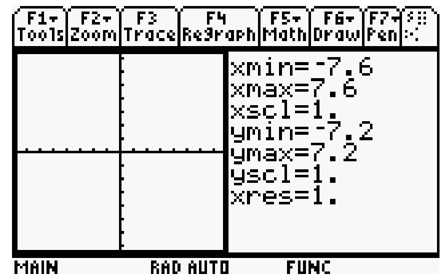

To move between screens, 2nd APPS

After you have set the window as indicated above, go to the Y= screen on the right of your split screen. Enter functions in the Y= screen, then press 2nd APPS to see their graphs.

Make these parabolas: