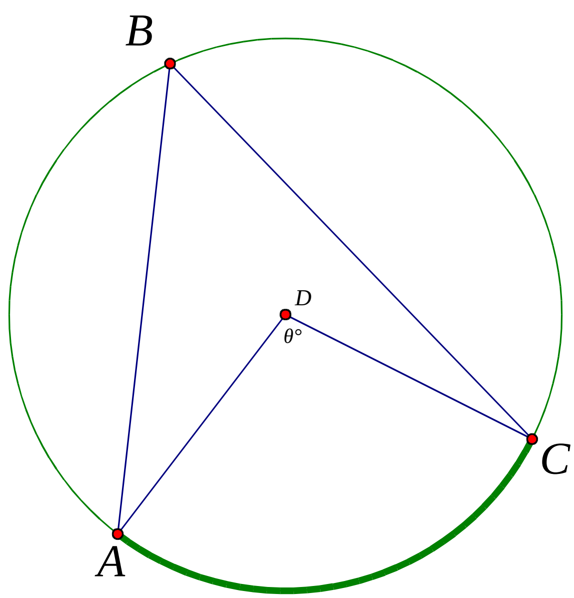
Name: ______________________________________ **Date:**_________________________

Triangles and Iteration

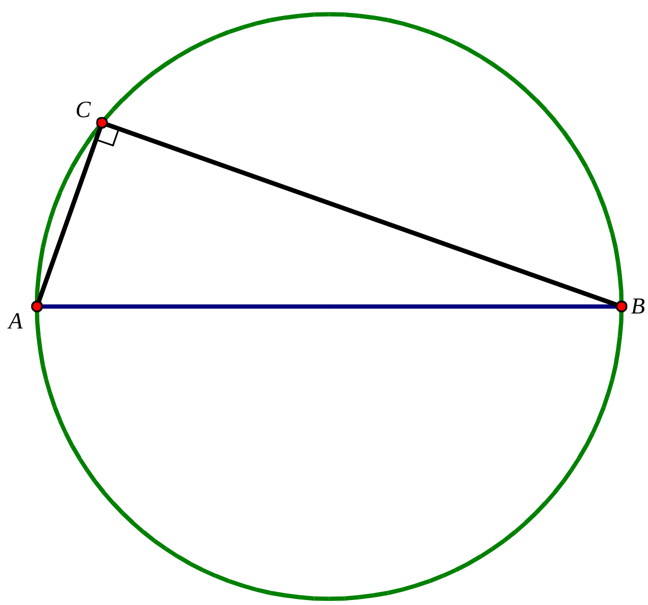
By Rachel Chou

Getting Ready

1. ∠ABC is called an inscribed angle, because its vertex is on circle D. Find θ in terms of ∠ABC.

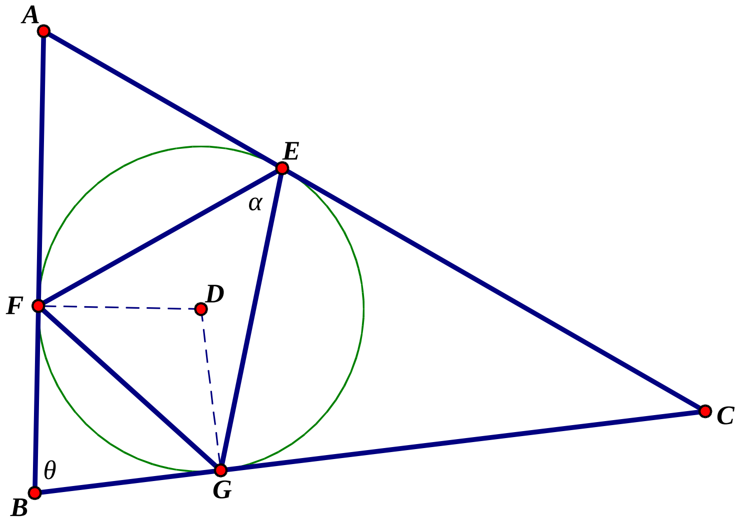


2. In the circle shown below, AC = 10 and BC = 24. Find the area outside the triangle but inside the circle. (This problem is connected to problem #1. If you see no connection after you finished, it either means your answer is wrong, or you made an assumption rather than having a good reason for something to be true. Please see if you see a connection between this problem and #1 before moving on.

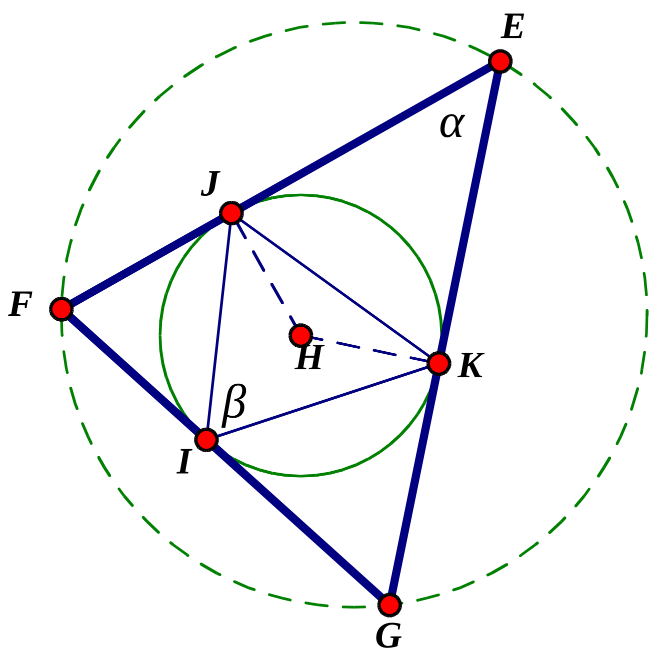


Inscribing Triangles

3. I have inscribed circle D in ΔABC shown below. The points of tangency are points E, F, and G, as shown. I then constructed ΔEFG. Find α, in terms of θ.



4. Inscribing circles inside of triangles seemed fun, so I did it again. That is, I took ΔEFG from above, and inscribed a circle H in side of it. The points of tangency were J, K, and I, as shown. I then created ΔJKI, as shown. Find β in terms of α, and then find β in terms of θ.



5. Now, let's say we do this over and over and over. Open up a spreasheet. Choose any arbitrary starting angle θ, and use the spreadsheet to help you find what the next 10 or so angles will be if you continue this process. If you do not know how to do this, get help from a classmate or your teacher.

Punch Lines

Punch line #1: If you continue this process over and over and over, the resulting triangle will approach an

________________________________ triangle. Your work in #3 demonstrates this fact. Formally prove it by

showing that for any arbitrary starting angle θ, the n^{th} iteration of this angle approaches a __________angle.

Punch line #2: Let's say that you have a sequence of numbers with an arbitrary starting value: a_1, and that $a_n = P - \frac{a_{n-1}}{k}$, where P>0 and k>1. Evaluate: . $\lim_{n \to \infty} a_n$