

Iterating the Logistic Equation

$$y=rx(1-x) \text{ with } r>0$$

Many properties of the iteration of the logistic equation can be investigated without technology.

1. Explain why a value of $x_n > 1$ leads to a negative value of x_{n+1} .
2. How many fixed points are there? Find a formula for each, as a function of r if necessary.
3. (For Calculus students) For what values of r is the entire curve under the $y=x$ line in the domain $[0,1]$? (Hint: for what value of r is the curve tangent to the $y=x$ line at the origin?)
4. Are your answers to the previous two questions consistent?
5. Where is the vertex? Find a formula for its coordinates as a function of r .
6. For what values of r is the vertex outside the unit square?
7. What are the implications for iterating the function in those cases?
8. Do an orbit analysis for the logistic equation. Consider only seeds in the interval $[0,1]$. See how much you can answer by studying the figures below, and then use technology. Consider the following cases:

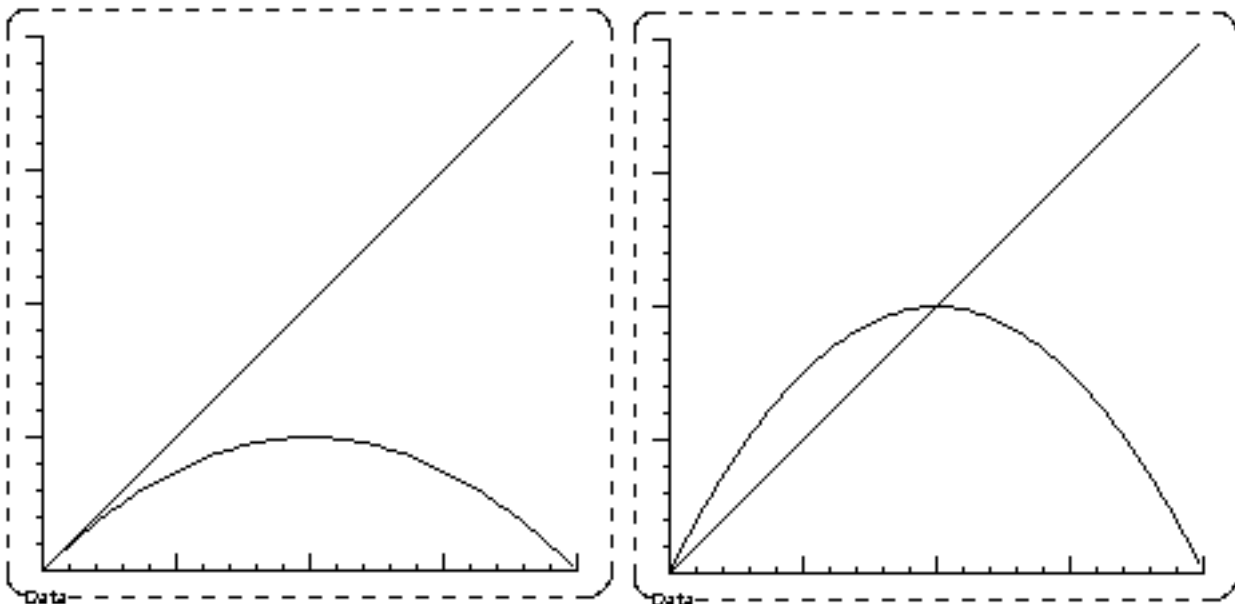
a. $0 < r < 1$	b. $r = 1$	c. $1 < r < 2$
d. $r = 2$	e. $2 < r < 3$	f. $r = 3$
g. $3 < r < 4$	h. $r = 4$	i. $r > 4$

Graphs with Various Values of r

Unit square: $x_{\min}=0, x_{\max}=1, y_{\min}=0, y_{\max}=1$.

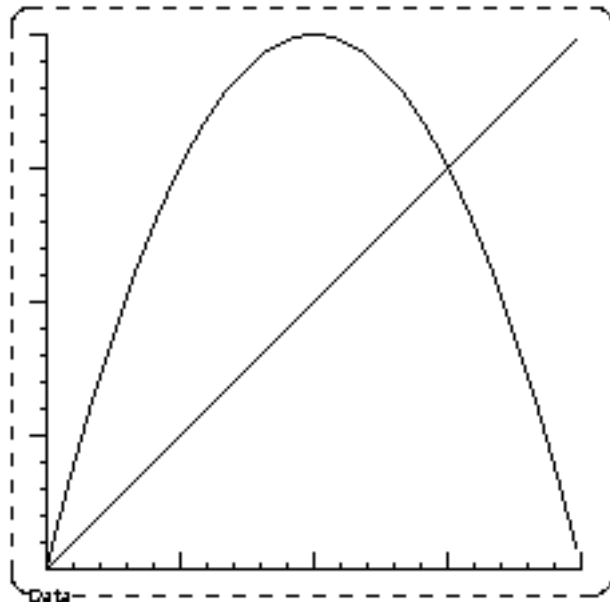
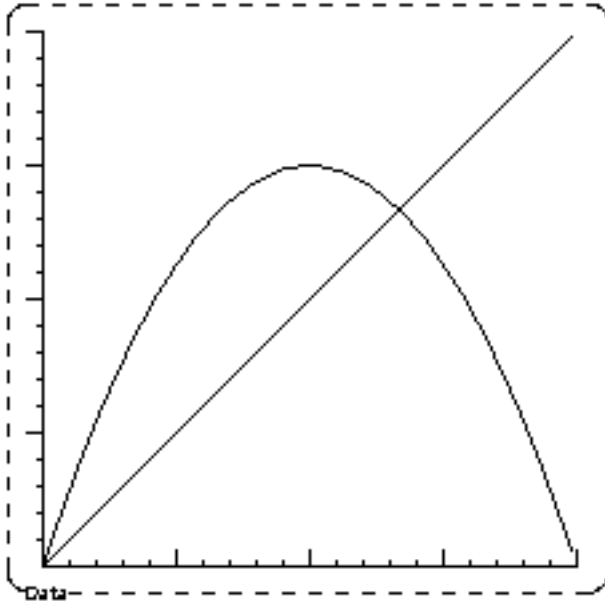
$r=1$

$r=2$



$r=3$

$r=4$



$r=5$ (this is zoomed out a bit, the unit square is marked)

