LESSON 14.3

Fractions and Equations

1. **Exploration** Wanda always enjoyed the math tests Mr. Stevens gave every Friday. She especially liked the tests on fractions. Here is the test she took on Friday the 13th. Try to find the problems she did wrong. If necessary, substitute numbers. If you can, show her how to do them correctly.

a. $\frac{2x}{5} - \frac{x}{3} = \frac{x}{2}$

b. $\frac{x}{5} + \frac{x}{5} = \frac{2x}{10}$

c. $\frac{x}{5} \cdot \frac{x}{5} = \frac{x^2}{25}$

d. $\frac{2x}{5} \cdot \frac{5}{2x} = 1$

e. $\frac{x}{5} + \frac{5}{x} = 2$

f. $\frac{M}{5} = \frac{10M}{50}$

g. $\frac{2M + 4}{M + 2} = 2$

COMPLICATING FRACTIONS

Sometimes it is useful to *complicate* fractions instead of simplifying them. For example, here are some more complicated fractions that are equivalent to $\frac{2x}{5}$.

a. $\frac{4x^2}{10x}$ b. $\frac{2xy}{5y}$ c. $\frac{8x + 2x^2}{20 + 5x}$

2. What was $\frac{2x}{5}$ multiplied by to give each one of the fractions? Sketch a Lab Gear fraction for part (a).

3. Write three fractions that are equivalent to $2/(x - 3)$. Check the correctness of a classmate's fractions.

4. Write a fraction that is equivalent to $\frac{x + 2}{5}$ that has the following:

a. a denominator of 10

b. a denominator of $5x + 15$

c. a numerator of $4x + 8$

d. a numerator of $3x^2 + 6x$

5. If possible, write a fraction that is equivalent to $\frac{y + x}{4x}$ that has the following:

a. a denominator of $8xy$

b. a denominator of $6x^2$

c. a numerator of $-2y - 2x$

d. a numerator of $3y + x$

6. Write a fraction equivalent to 2 that has $5a^2$ as a denominator.

7. Write a fraction equivalent to 1 that has b as a denominator.

8. Write a fraction equivalent to b that has b as a denominator.

9. Write a fraction equivalent to x that has x^2 as a denominator.

COMMON DENOMINATORS

To add or subtract fractions having unlike denominators, you first have to find a common denominator.

10. a. Write a fraction equivalent to $\frac{b}{3}$ having a denominator of $6a^2$.

b. Add $\frac{b}{3} + \frac{c}{6a^2}$.

11. Write two fractions whose sum is $\frac{2x + 5}{10x}$.

12. a. Write a fraction equivalent to $\frac{bc}{5a}$ having a denominator of $5ac$.

13. Find a common denominator and add or subtract.

a. $\frac{1}{4x} + \frac{1}{10x^2}$ b. $\frac{5}{xy} - \frac{1}{x^2}$

FROM QUADRATICS TO FRACTIONS

Tara was trying to solve $x^2 + 4x - 6 = 0$ with the zero product property. She couldn't figure out a way to factor the trinomial. Then she had an idea. She wrote:

$$x^2 + 4x = 6$$
$$x(x + 4) = 6$$

Tara was still thinking about the zero product property. She wrote:

$$x = 6 \text{ or } x + 4 = 6$$

14. Explain why Tara's reasoning is incorrect. (Why does this method work when one side of the equation is 0?)

When Tara saw her mistake, she tried another method. She divided both sides by x.

$$x(x + 4) = 6$$
$$x + 4 = \frac{6}{x}$$

Then she was stuck. Her teacher suggested that she use trial and error, so she made this table.

x	$x + 4$	$\frac{6}{x}$
1	5	6
2	6	3
1.5	5.5	4
1.25	5.25	4.8
1.13	5.13	5.31

15. Continue the table and find a value of x that, when substituted into both sides of the equation, will give the same value
a. to the nearest tenth;
b. to the nearest hundredth.

16. The quadratic equation that Tara was solving has two roots. Approximate the other root to the nearest hundredth.

17. Solve the equation $x^2 + 5x - 3 = 0$ using trial and error. (You do not need to do it in the same way as Tara.) Approximate each solution to the nearest hundredth.

18. Confirm your solution by using a method you learned in Chapter 13.

FROM FRACTIONS TO QUADRATICS

Rewrite each equation as an equivalent quadratic equation. Then try to solve it. Show each step.

19. $x + 4 + \frac{3}{x} = 0$

20. $2m + \frac{4}{m} = 9$

21. $4x = \frac{1}{x}$

22. $L - 4 = \frac{20}{L}$

23. $\frac{1}{x} = x + 1$

24. $\frac{4}{x} = x + 1$