LESSON 9.1

Distance

You will need:

graph paper

TAXICAB DISTANCE

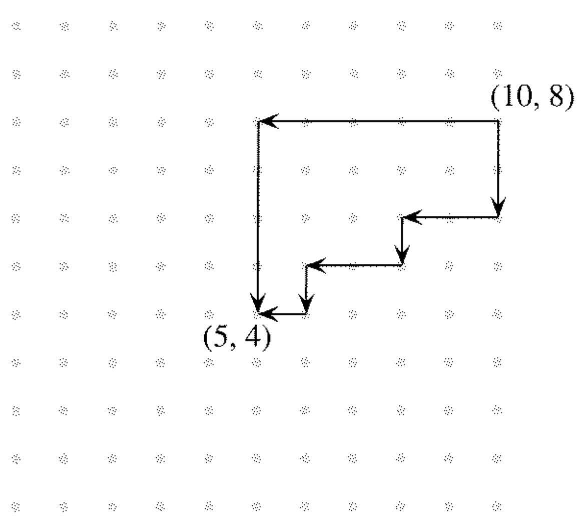
1. Assume you can travel only horizontally and vertically on the Cartesian plane, never letting your x- or y-coordinates decrease.

a. Find at least three ways to get from the origin to (3, 4).

b. Does the travel distance depend on the path you found in part (a) or is it the same for all of them? Explain.

Definition: The *taxicab distance* between two points in the Cartesian plane is the length of the *shortest path* between them that consists of *only* horizontal and vertical segments. Taxicab distance gets its name because it models distance in a city with a network of perpendicular streets.

Example: The taxicab distance from (10, 8) to (5, 4) is 9.



2. What is the taxicab distance between:

a. (1, 2) and (6, 7)?

b. (1, 2) and (1, 7)?

c. (1, 7) and (6, 2)?

d. (-1, -7) and (6, -2)?

e. (1.2, 3.4) and (5.67, 8.9)?

3. a. Find all the points that are at taxicab distance 5 from (5, 5). Sketch them.

b. Describe the shape you found in part (a). Some math teachers call this shape a *taxicab circle*. Explain why.

c. What else might this shape be called?

4. Describe the set of points whose taxicab distance from (5, 5) is

a. greater than 5;

b. less than 5.

TAXICAB vs. EUCLIDEAN DISTANCE

Euclidean distance (named after the ancient Greek mathematician Euclid) is the straight-line distance ("as the crow flies") we studied in a previous lesson.

5. A crow and a taxicab go from the origin to (5, 5). How far does each have to travel?

6. Give examples, if possible, and explain.

a. When are Euclidean and taxicab distances between two points equal?

b. When is Euclidean distance greater than taxicab distance?

c. When is taxicab distance greater?

7. *A straight line is the shortest path between two points.* Explain how this statement is relevant to problem 6.

8. Sketch all the points that are at the same taxicab distance from both (4, 3) and (6, 7).

9. Sketch all the points that are at the same Euclidean distance from both (4, 3) and (6, 7).

10. Find all points P such that:
   - the taxicab distance from P to (4, 3) is greater than the taxicab distance from P to (6, 7), **but**
   - the Euclidean distance from P to (6, 7) is greater than the Euclidean distance from P to (4, 3).

   Explain, using sketches and calculations.

ABSOLUTE VALUE

11. Find the Euclidean distance between:
   a. (1, 2) and (6, 2);
   b. (6, 2) and (1, 2);
   c. (6.7, 3.45) and (8.9, 3.45).

12. Explain in words how to find the distance between (x_1, y) and (x_2, y) if:
   a. $x_1 > x_2$; b. $x_1 < x_2$.

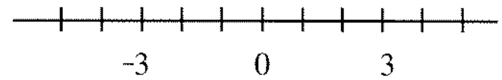
If the y-coordinates of two points are the same, the distance between the two can be found by subtracting the x-coordinates. If the result of the subtraction is negative, use its opposite, since distance is always positive. This is called the *absolute value* of the difference.

13. Find the absolute value of the difference between:
   a. 2 and 5; b. 3 and -9;
   c. -2 and -5; d. -3 and 9.

14. Explain how you find the distance between two points whose x-coordinates are the same. Give an example.

Definition: The *absolute value* of a number x is the distance from x to 0 on the number line.

Example: The absolute value of 3 is 3. The absolute value of -3 is also 3.



15. Find the absolute value of:
   a. 12; b. -1/4.

Notation: The absolute value of a number z is written $|z|$. For example:

$|2| = 2$ $\qquad |-2| = 2$

The absolute value of a difference can be written using the same symbol. For example, the absolute value of the difference between a and b is written $|a - b|$.

16. Find the absolute value of:
   a. $3 - -5$; b. $-5 - 3$.

17. What is the distance between x and 3 on the number line?
   a. Explain in words how to find it.
   b. Write a formula, using absolute value notation.

18. Using absolute value notation, the distance between (x_1, y) and (x_2, y) can be written $|x_1 - x_2|$ or $|x_2 - x_1|$. Explain.

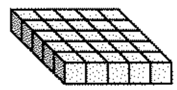
19. Use absolute value notation to write the distance between (x, y_1) and (x, y_2).

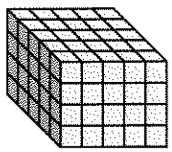
20. Use absolute value notation to write the taxicab distance between (x_1, y_1) and (x_2, y_2).

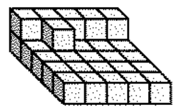
REVIEW SURFACE AREA OF BUILDINGS

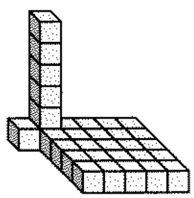
Find the volume and surface area of each of these buildings (including the underside).

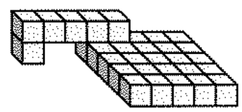
21. 

22. 

23. 

24. 

25. 

26. 

PREVIEW MIXTURES

Tina was thirsty, so Tina and Lana decided to make lemonade. They planned to make a lot, so they could sell some of it at a roadside stand.

Tina started making lemonade using the "taste" method. She added 21 cups of water to 16 cups of lemonade concentrate, but it tasted too lemony. Then she noticed directions on the lemonade package:

> Add water to taste. Most people like a mixture that is 1/5 to 1/4 concentrate.

27. How much water should she add to get a mixture that is 1/5 concentrate?

28. Lana tasted the lemonade after Tina had added water to get a mixture that was 1/5 concentrate. It didn't taste lemony enough. How much lemonade concentrate should they add now to get a mixture that is 1/4 concentrate?