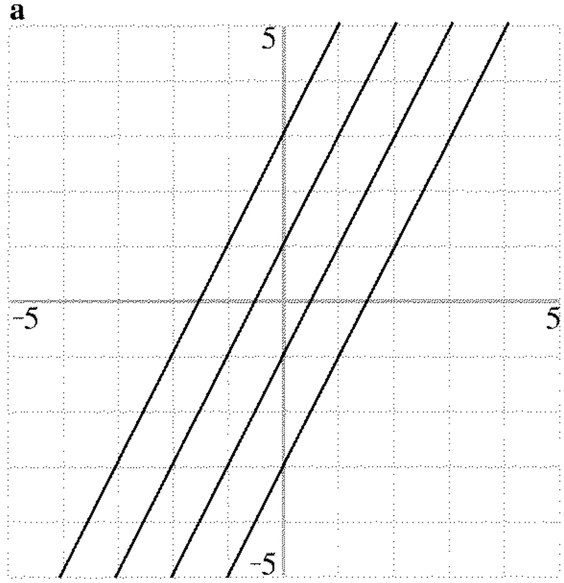
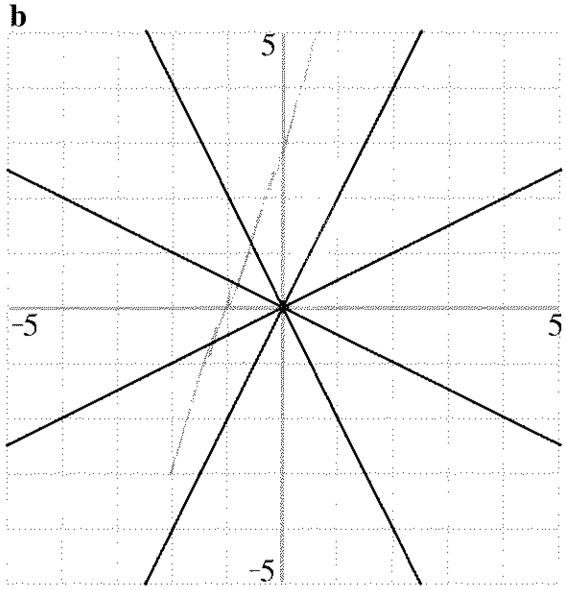
LESSON 8.4

Linear Functions

You will need: graph paper

1. **Exploration** For problems (a-b), find the equations of lines that will create the given design.





THE SLOPE OF A LINE

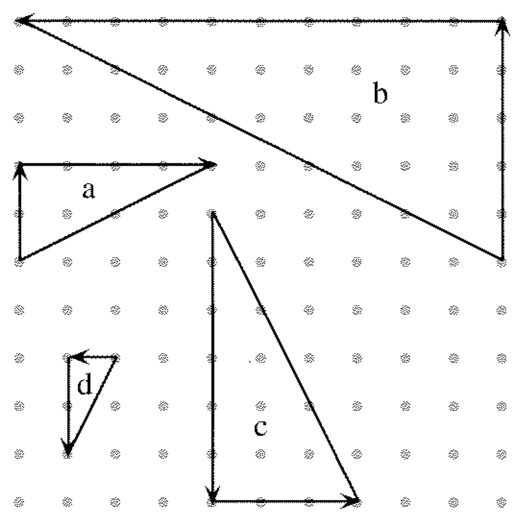
Definitions: The *rate of change* of a function is defined as a ratio between the change in y and the change in x.

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

In the Cartesian plane, a change in y-coordinates is called a *rise.* A change in x-coordinates is called a *run.*

The *slope of a line* is the ratio obtained when you divide the rise by the run. If you move from left to right, the run is positive. From right to left, it is negative. If you move up, the rise is positive. Moving down, it is negative.

The figure shows right triangles for slopes 0.5, -0.5, 2, and -2.



2. Match the slope to the triangle by finding the rise and the run as you move from one end of the hypotenuse to the other
   a. in the direction of the arrows;
   b. in the opposite direction.
   c. Do you get the same answers both ways?

3. What can you say about the slope of a line if, when you follow the line *from left to right,*
   a. it goes up?
   b. it goes down?
   c. it goes neither up nor down?
4. Find two (x, y) pairs that satisfy each equation. Use them to graph the line. Label the two points, and use them to find the slope.
   a. $y = 1.5x + 3$ b. $y = -1.5x + 3$
   c. $y = 2x + 3$ d. $y = -3x + 3$
5. Think of the line with equation $y = 3x + 3$.
   a. Predict its slope.
   b. Check your prediction by graphing.
   c. For this function, when x increases by 1, by what does y increase?
6. Repeat problem 5 for $y = -2x + 3$.
7. How is the coefficient of x related to the slope?

THE y-INTERCEPT OF A LINE

8. For each of these equations, find the y-intercept.
   a. $y = 0.5x + 3$
   b. $y = 0.5x - 3$
   c. $y = 0.5x$
   d. $y = 0.5x + 1.5$

One way to find the y-intercept of a function is to graph it, and see where the graph meets the y-axis. Another way is to remember that *on the y-axis, the x-coordinate is 0.* In other words, all points on the y-axis are of the form (0, ___). So to find the y-intercept of a function, it is enough to substitute 0 for x, and find the value of y.

For each of these linear functions, answer the following questions. Graph the functions if you need to check your answers.
   a. When $x = 0$, what is y?
   b. When x increases by 1, by how much does y increase? (If y decreases, think of it as a negative increase.)
   c. What are the slope and y-intercept?

9. $y = x + 2$ 10. $y = -4 - 3x$
11. $y = -x$ 12. $y = 9$
13. $y = \frac{6x - 7}{8}$ 14. $y = -2(x - 3)$

SLOPE AND y-INTERCEPT

Definition: $y = mx + b$ is called the *slope-intercept form* for the equation of a line.

For each equation below, tell whether it is in slope-intercept form.
   a. If it is, name m and b.
   b. If not, put it in slope-intercept form, then name m and b.

15. $y = 5x - 6$ 16. $y = -4(x - 7)$
17. $y = \frac{5x - 6}{3}$ 18. $y = \frac{x - 7}{-4}$
19. $y = 3(5x - 6)$ 20. $y = -4x - 7$
21. $y + 4 = x$ 22. $y + x = 4$
23. Without graphing each pair of lines, tell whether or not their graphs would intersect. Explain.
   a. $y = 2x + 8$ $y = 2x + 10$
   b. $y = -2x + 8$ $y = 2x + 10$
   c. $y = -2$ $y = 10$
   d. $y = x/4$ $y = 0.25x + 10$
   e. $y = 2(5x - 3)$ $y = 10x$

24. For (a-c), give the equation of a line that satisfies the following conditions.

a. It passes through the point (0, -2), and goes uphill from left to right.

b. It passes through the origin and (4, -6).

c. It does not contain any point in the third quadrant, and has slope -1.5.

Compare your answers with your classmates' answers.

25. Write three equations of the form $y = mx + b$. For each one, tell how much x changes when y changes by:

a. 1; b. 5; c. K.

26. Did your answers to problem 25 depend on the parameter m, the parameter b, or both?

27. What can you say about the signs of the slope and y-intercept of a line that does not contain any points in:

a. the first quadrant?

b. the second quadrant?

c. the third quadrant?

d. the fourth quadrant?

28. Report Explain how to use the slope-intercept form to predict the slope and y-intercept of a line. Make sure you give examples as you answer the following questions.

- What is the value of y when $x = 0$?
- When x increases by 1, by how much does y increase?
- How about when x increases by d?
- If two lines are parallel, what do their equations have in common?
- If two lines meet on the y-axis, what do their equations have in common?
- How is the slope-intercept form useful for graphing lines quickly?

PREVIEW ***WHAT'S THE FUNCTION?***

29. Think of the line that has slope -2 and passes through (1, 4).

a. By graphing, find any other point on the line.

b. Look at the graph to find the y-intercept.

c. What is the equation of the line?

30. Graph the line $y = 2x - 5$. Then graph each line, (a-c), and find its slope, y-intercept, and equation.

a. any line parallel to $y = 2x - 5$

b. the line parallel to it that passes through the origin

c. the line parallel to it that passes through the point (1, 4)