LESSON 8.2

Focus on Function Diagrams

You will need:

graph paper

REVIEW PARALLEL-LINE DIAGRAMS

1. a. Draw a function diagram such that its in-out lines are *parallel* and going uphill (from left to right).
   b. Find the function corresponding to the diagram, using an in-out table if you need it.

2. Repeat problem 1 with parallel in-out lines going
   a. downhill;
   b. horizontally.

3. For the functions you created in problems 1 and 2, when x increases by 1, by how much does y increase? Does it depend on the steepness of the lines?
   (To answer this, compare your functions with other students' functions.) Explain your answer.

Problems 4 through 9 refer to the function diagrams shown on the next page.

THE FOCUS

Definition: If an in-out line is horizontal, its input is called a *fixed point.*

For example, both x and y equal 12 in diagram (a), so 12 is a fixed point for that function.

4. What are the fixed points for functions (b-p)?

Definition: In-out lines can be extended to the left or right. If all of them meet in a single point, that point is called the *focus.*

5. **Exploration** Consider the function diagrams shown in figures (a-p). For each one, find the function. You may split the work with other students. Describe any patterns you notice. If you cannot find all the functions or patterns, you will get another chance at the end of the lesson.

MAGNIFICATION

6. Look at function diagram (h). By how much does y change when x increases by:
   a. 1? b. 2?
   c. some amount A?

In function diagrams that have a focus, changes in y can be found by multiplying the changes in x by a certain number, called the *magnification.*

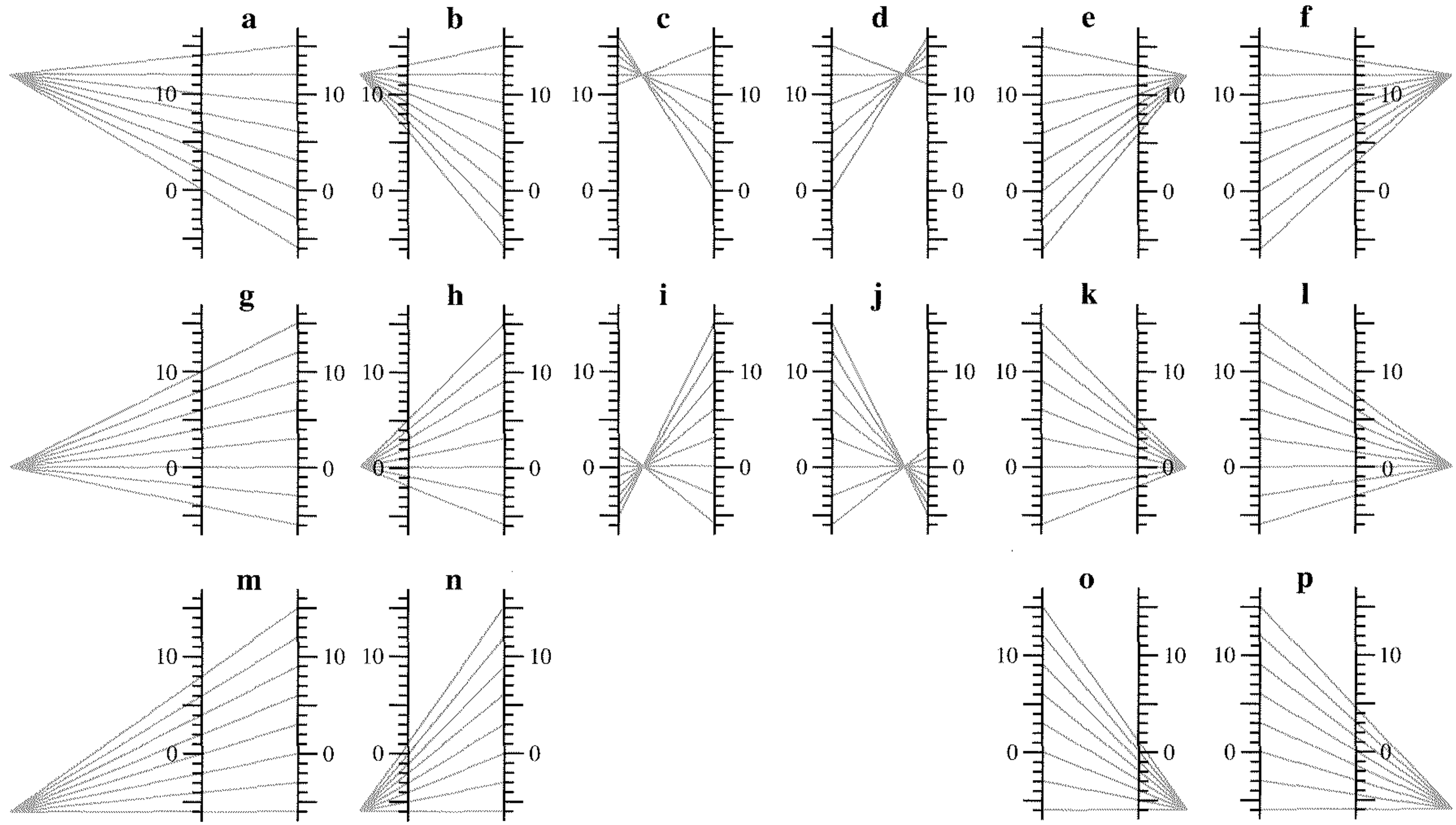
(change in x) · (magnification) = (change in y)

7. a. What is the magnification for (h)?
   b. What other diagrams have the same magnification?

Rule: If y decreases when x increases, the magnification is negative.

8. For which diagrams is the magnification equal to -3? (If x increases by 1, y *decreases* by 3.)

9. Find the magnification for each function diagram. Note that the magnification can be positive or negative, a whole number or a fraction.



sixteen function diagrams

THE m PARAMETER

You probably noticed that all the function diagrams represent functions of the form $y = mx + b$. It turns out that this is always true of function diagrams with a focus. As you may remember, the letters m and b in the equation are called *parameters.*

10. Look at the equations you found in the Exploration, problem 5. What is the relationship between the magnification and the m parameter in those equations? Explain.

11. If you move the focus of a function diagram up, how does it affect the value of m? How about if you move it down?

12. Where would the focus be if m was

a. a negative number?

b. a number between 0 and 1?

c. a number greater than 1?

13. What is a possible value of m if the focus is

a. half-way between the x- and y-number lines?

b. between the x- and y-number lines, but closer to x?

c. between the x- and y-number lines, but closer to y?

14. What is a possible value of m if the focus is

a. far to the left of the x-number line?

b. close to the left of the x-number line?

c. close to the right of the y-number line?

d. far to the right of the y-number line?

15. In some parts of mathematics, parallel lines are said to meet at a point that is *at infinity.* In that sense, parallel-line diagrams could be said to have a focus at infinity. Is this consistent with your answer to problem 14? Explain.

RATE OF CHANGE

Once again, look at the diagrams (a-p).

16. On each diagram, *as x increases,* follow y with your finger. For what values of m does y

a. go up? b. go down?

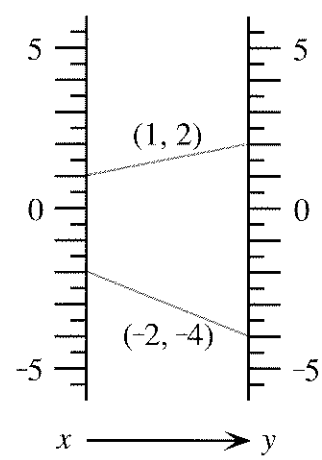
c. move fast? d. move slowly?

The magnification is often called the *rate of change.*

17. What is the rate of change if y increases by 3 when x increases by:

a. 1? b. 6? c. -10?

THE b PARAMETER



Two in-out lines are shown in the diagram. Each one is labeled with a number pair. The first number in the pair is the input, and the second number is the output.

Notation: Any in-out line can be identified by a number pair. From now on, we will refer to lines on function diagrams this way. For example, the line connecting 0 on the x-number line to 0 on the y-number line will be called the (0, 0) line.

18. What can you say about the b parameter if the focus is on the (0, 0) line?

19. Look at diagram (n). Its equation is $y = 3x + 12$.

a. Name the in-out lines that are shown.

b. Check that the pairs you listed actually satisfy the equation by substituting the input values for x.

c. Among the pairs you checked was (0, 12). Explain why using 0 as input gave the b parameter as output.

20. In most of the diagrams (a-p), there is an in-out line of the form (0, ___). How is the number in the blank related to the b parameter? Explain.

$y = mx + b$

21. If you did not find all the equations for the function diagrams (a-p), when working on problem 4, do it now. Hint: You may use what you learned about magnification and about the (0, ___) in-out line.

22. **Summary** Write what you learned about function diagrams, the fixed point, the focus, magnification, and the parameters m and b. Also mention parallel-line diagrams.

REVIEW *BINOMIAL MULTIPLICATION*

Multiply and combine like terms.

23. $(3x + 1)(x - 2)$

24. $(2x - 3)(5 - x)$

25. $(5 + x)(3x - 3)$

26. $(2y - 2)(6 - y)$

27. $(3x - 1)(2 + x)$

28. $(3x + 1)(2 + x)$

29. $(6 + y)(2y + 4)$

30. $(y - 4)(2y + 2)$

31. $(y - 3)(y - 5)$

32. $(6 - x)(2x - 3)$