LESSON 7.1

Squares and Cubes

You will need:

the Lab Gear 

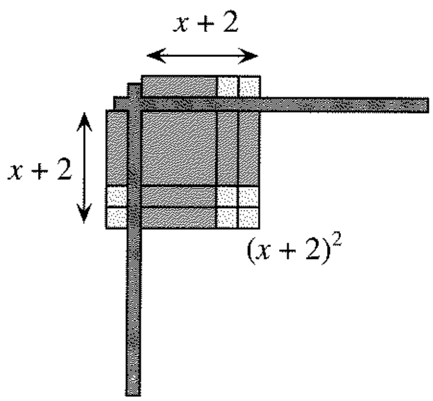
1. **Exploration** Which is greater, $2^2 + 3^2$ or $(2 + 3)^2$? By how much? Which is greater, $5^2 + 8^2$ or $(5 + 8)^2$? By how much? Is it ever true that

$$x^2 + y^2 = (x + y)^2 ?$$

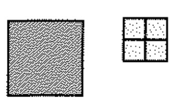
How far apart are they? Experiment and write a paragraph summarizing your work and your conclusions. It may help to use the Lab Gear.

HOW MANY SQUARES?

The square $(x + 2)^2$ can be written as the product $(x + 2)(x + 2)$. It can be represented by *a single square* with side $(x + 2)$, as shown in the figure.



The sum of the squares $x^2 + 2^2$ cannot be written as a product or represented with a single square. It must be represented by *two individual squares.*



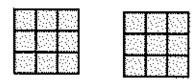
$x^2 + 2^2$

Compare these two expressions.

(i) $2 \cdot 3^2$

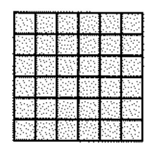
(ii) $(2 \cdot 3)^2$

Because the rules for order of operations tell us to perform exponentiation first, expression (i) means *square 3 and then multiply by 2.* This can be modeled by building two squares with the Lab Gear.



$2 \cdot 3^2$

Expression (ii) means *multiply 2 by 3 and square the result.* Since $2 \cdot 3 = 6$, this can be written more simply as 6^2. This can be modeled by building *one* square with the Lab Gear.



6^2

Make a rough sketch representing each expression, 2-6, with as few squares as possible. Which of these expressions can be modeled as a single square? Which require more than one square? (Be careful!)

2. a. $(x + 1)^2$ b. $x^2 + 1$

3. a. $4x^2 + 4$ b. $(2x + 2)^2$

4. a. $5^2 + 3 \cdot 5^2$ b. $2^2 + 5 \cdot 2^2$

5. a. $3^2 + 4^2$ b. $(3 + 4)^2$

6. a. $(3 \cdot 4)^2$ b. $3^2 \cdot 4^2$

7. Give the value of each expression.

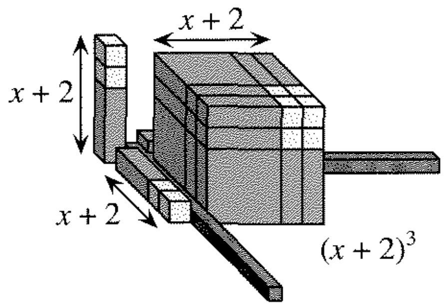
a. $3^2 + 4^2$ b. $(3 \cdot 4)^2$

c. $(3 + 4)^2$ d. $3^2 \cdot 4^2$

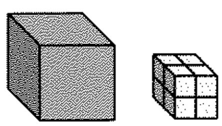
e. $5^2 + 3 \cdot 5^2$ f. $2^2 + 5 \cdot 2^2$

HOW MANY CUBES?

The cube $(x + 2)^3$ can be written as the product $(x + 2)(x + 2)(x + 2)$. It can be represented by *a single cube* with sides $(x + 2)$, as shown.



The sum of the cubes $x^3 + 2^3$ cannot be written as a product. It cannot be represented with a single cube. It must be represented by *two individual cubes.*

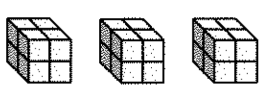


$x^3 + 2^3$

Compare these two expressions.

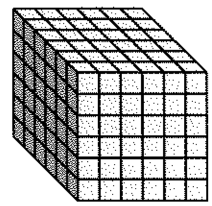
(i) $3 \cdot 2^3$
(ii) $(3 \cdot 2)^3$

Because the order of operations tells us to perform exponentiation first, expression (i) means *cube 2 and then multiply by 3.* This can be modeled by building three cubes with the Lab Gear.



$3 \cdot 2^3$

Expression (ii) means *multiply 2 by 3 and cube the result.* Since $3 \cdot 2 = 6$, this can be written more simply as 6^3. This can be modeled by building one cube with the Lab Gear.



6^3

8. What number does each expression equal?
a. $3 \cdot 2^3$ b. $(3 \cdot 2)^3$

How would you represent each expression with as few cubes as possible? It may help to use the Lab Gear. Make a sketch, giving the dimensions of each cube.

9. a. $(x + 1)^3$ b. $x^3 + 1$

10. a. $x^3 + 8$ b. $(x + 2)^3$

11. a. $x^3 + y^3$ b. $(x + y)^3$

Which of these expressions could be modeled using only one cube? Which require more than one cube? Tell how you would represent each expression with as few cubes as possible. Give the dimensions of each cube.

12. a. $6 \cdot 2^3$ b. $(6 \cdot 2)^3$

13. a. $6^3 + 2^3$ b. $(6 + 2)^3$

14. What is the value of each expression?
a. $6 \cdot 2^3$ b. $(6 \cdot 2)^3$
c. $6^3 + 2^3$ d. $(6 + 2)^3$

MAKING SQUARES FROM CUBES

15. a. Use the Lab Gear to show how the expression $1^3 + 2^3 + 3^3$ can be modeled by building three cubes.

b. What was the total number of blocks needed for part (a)?

c. Make a square by rearranging the blocks you used to make the three cubes. What are the dimensions of the square?

16. a. The expression $1^3 + 2^3 + 3^3 + 4^3$ could be modeled by building four cubes. What is the total number of blocks used?

b. How would one make a square by rearranging these blocks? Give the square's dimensions.

17. Compare your answers to problems 15 and 16. Look for a pattern. Check it for $1^3 + 2^3$. Predict the value of the sum,

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3.$$

Check your prediction.

18. **Generalization** The expression

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ... + n^3$$

can be modeled by building n cubes out of blocks. Could you rearrange these blocks into a square? If so, what would its dimensions be? Explain your answer.

REVIEW *CUBING WITH A TABLE*

To find the cube of a polynomial, first find its square, then multiply the result by the polynomial. For example, to calculate $(x + 2y)^3$, first square $x + 2y$.

	x	$2y$
x	x^2	$2xy$
$2y$	$2xy$	$4y^2$

Combine like terms in the body of the table. Multiply this result by $x + 2y$.

	x^2	$4xy$	$4y^2$
x	x^3	$4x^2y$	$4xy^2$
$2y$	$2x^2y$	$8xy^2$	$8y^3$

So $(x + 2y)^3 = x^3 + 6x^2y + 12xy^2 + 8y^3$.

19. Find the cube.

a. $(x + 1)^3$ b. $(2x + 2)^3$

c. $(x + y)^3$ d. $(2x - y)^3$

e. $(3x + 2y - 5)^3$