LESSON 6.4

Equations and Identities

You will need:

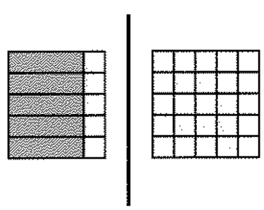
the Lab Gear 

Definition: An *identity* is an equation that is true for all values of the variables.

1. Which of these equations are identities? Explain your answers.
   a. $3x + 9 - 2(x + 2) = 3x + 9 - 2x + 2$
   b. $3x + 9 - 2(x + 2) = 3x + 7(x + 2)$
   c. $3x + 9 - 2(x + 2) = x + 5$
   d. $3x + 9 - 2(x + 2) = x + 7$

USING THE LAB GEAR

To solve the equation $5(x + 1) = 25$ you can model both the left side and the right side as rectangles. In this case, you can match the rectangles, and it is easy to see what the value of x must be.



2. What is the value of x that makes both sides equal?

Use the Lab Gear to solve these equations. If the equation is an identity, explain how you know, using sketches if necessary.

3. $3(x + 2) = 15$
4. $3(x + 2) = 3x + 6$
5. $4(2x + 1) = 4(x + 5)$
6. $4(2x - 1) = 4(x - 1)$
7. $4(2x - 1) = 4(2x + 1)$
8. $2(2x + 2) = 4(x + 1)$
9. $4(2x - 2) = 2(4x - 4)$

USING GRAPHS AND TABLES

10. Make a table of (x, y) pairs and graph each linear function.
    a. $y = -2(x - 1) + 2$
    b. $y = -2x + 4$
11. By simplifying the left side, show that $-2(x - 1) + 2 = -2x + 4$ is an identity.
12. For each pair of functions, decide whether or not both members of the pair would have the same graph. Explain.
    a. $y = 3 - 4x$ and $y = 4x - 3$
    b. $y = -6 - 8x$ and $y = 8x - 6$
    c. $y = 2x^2$ and $y = 2x(x + 2) - 4x$
    d. $y = 5 - x$ and $y = -x - 5$
    e. $y = -x + 5$ and $y = 5 - x$
13. Look at your answers to problem 12. For each pair that would not have the same graph, graph both functions on the same axes. Find the point where the two graphs intersect and label it on the graph.
14. Which of the pairs of graphs that you drew in problem 13 do not have a point of intersection? Can you explain why this is so?
15. When graphing two linear functions, there are three possibilities: You may get the same line, two parallel lines, or two lines that intersect. Explain what the tables of (x, y) values look like in each case.

ALWAYS, SOMETIMES, NEVER

While an identity is true for all values of x, an equation may be true for only some values of x, or for no values of x.

Examples: $2x + 6 = 4$ is true when $x = -1$, but not when $x = 0$. The equation $x + 5 = x$ is never true, because a number is never equal to five more than itself. We say this equation has *no solution.*

16. For each equation, state whether it is *always, sometimes,* or *never* true. If it is always or never true, explain how you know. It may help to simplify and to use tables, graphs, or sketches of the Lab Gear.

a. $2x + 5 = 2x + 1$

b. $3(x - 4) - 4(x - 3) = 0$

c. $(x + 5)^2 = x^2 + 25$

d. $6x - (7 - x) + 8 = 7x + 1$

17. Look at the equations in problem 16 that you decided were *sometimes* true. For each one, find a value of x that makes it true and one that makes it false. Show your work.

For each equation 18-21:

State whether the equation is always, sometimes, or never true. Explain.

18. $0.5x - 2 = 0.5(x - 2)$

19. $0.5x - 2 = 0.5(x - 4)$

20. $0.5x - 2 = x - 4$

21. $0.5(x - 2) = x - 4$

22. Report Write a report about equations that are always, sometimes, or never true. Use one example of each type. Illustrate each example with a graph and a Lab Gear sketch. Be sure to include the definition of *identity* and full explanations.

REVIEW ***WHICH IS GREATER?***

23. Which is greater, or does it depend on the value of x? Explain.

a.	$-2x$	$-2x + 7$
b.	$6x - 4$	$6x + 4$
c.	$-x^2$	x^2
d.	$(-x)^2$	$-x^2$

REVIEW/PREVIEW ***MAKE A SQUARE***

Make a square with these blocks, adding as many yellow blocks as you want, but nothing else. For each square, write an equation relating the side length to the area.

24. $x^2 + 10x +$ __

25. $4x^2 + 8x +$ __

26. $9x^2 + 6x +$ __

27. $x^2 + 2x +$ __

28. $4x^2 + 12 +$ __

29. Is it possible to get a different square by adding a different number of yellow blocks? Explain your answer.

Make a square with these blocks, adding as many x-blocks as you want, but nothing else. For each square, write an equation relating the side length to the area.

30. $x^2 +$ __ $+ 25$

31. $4x^2 +$ __ $+ 25$

32. $x^2 +$ __ $+ 36$

33. $9x^2 +$ __ $+ 1$

34. $x^2 +$ __ $+ 9$

35. Is it possible to get a different square by adding a different number of x-blocks? Explain your answer.

36. Summary Describe the pattern for the square of a binomial, in terms of the Lab Gear, and in terms of the algebraic symbols.