LESSON 5.6

Factors

You will need:

the Lab Gear 

1. Exploration

a. Draw a rectangle whose sides are any two consecutive even numbers, like 4 and 6. Find its area. If the side lengths have to be whole numbers, is it possible to draw a rectangle having the *same area but different sides*? Try this with another pair of consecutive even numbers. Is it possible this time? Do you think it is always, sometimes, or never possible?

b. Does your result change if you use two consecutive odd numbers, like 3 and 5?

c. What about consecutive multiples of 3, like 6 and 9?

SAME AREA, DIFFERENT PERIMETER

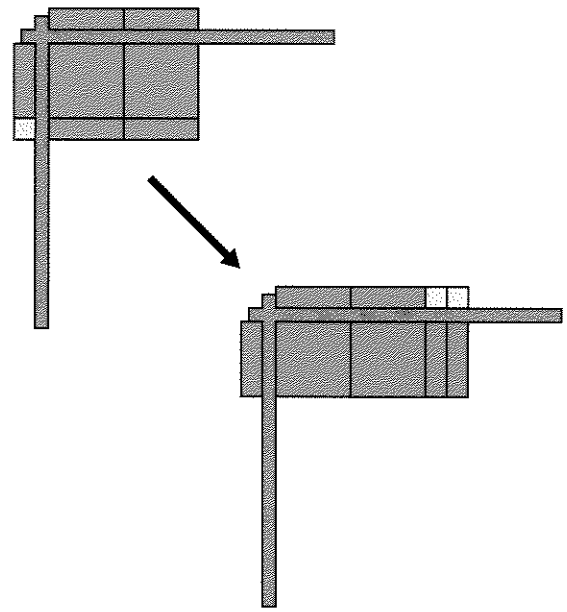
Example: Use the Lab Gear to build a rectangle having a width of $2x$ and a length of $x + 1$.

a. Sketch the rectangle. Label it with an equation of the form *length times width equals area.*

b. Find the perimeter of the rectangle.

c. Rearrange your rectangle into a rectangle having the *same area* but a *different perimeter.*

d. Write another equation of the form *length times width equals area.*



For problems 2-4 below, build a Lab Gear rectangle of the given width and length. Then follow the instructions in parts (a) through (d) in the example.

2. width: $2x$ length: $2x + 2$

3. width: $3x$ length: $3 + x$

4. width: x length: $4 + 4x$

For problems 5-6 follow the instructions in the example, but build at least two rectangles, and three if possible.

5. width: $4 + 2x$ length: $2 + 4x$

6. width: $2 + 2x$ length: $3 + 2x$

RECOGNIZING FACTORS

For each expression, 7-12, write as many different products equal to it as you can. Use only whole numbers. (In some cases, it may be helpful to use the Lab Gear to build rectangles and/or boxes.)

7. 24

8. $6y^2$

9. $(2x + 4)(3x + 6)$

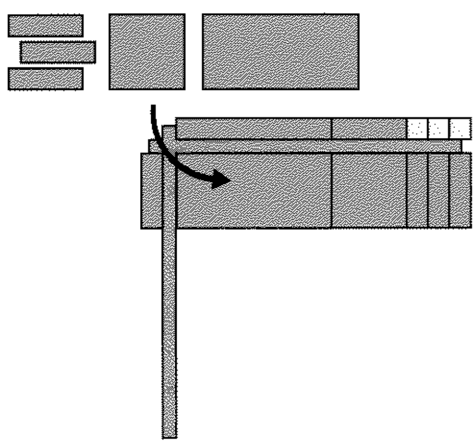
10. $12x^3$

11. $12x^2 + 4x$

12. $2x(6x + 18)$

COMMON FACTORS

Example: As you know, factoring a polynomial can sometimes be modeled by making a Lab Gear rectangle.



$$xy + x^2 + 3x = x(y + x + 3)$$

By multiplying the factors, you get the original polynomial back. Factoring is using the distributive law in reverse.

In this example, we say that x is a *common factor* of all three terms in the original polynomial, because it divides each term evenly. In the case of $2x^3 + 8x^2 + 2x^2y$, the common factors are 2, x, and x^2. In factoring such a polynomial, it is usually best to *take out* the *greatest common factor,* which is $2x^2$.

In the following problems, factor the polynomials by taking out the greatest common factor. Not all are possible.

13. $2x^3 + 8x^2 + 2x^2y$

14. $2x^2 - 6x$

15. $2x^2 + 6x + 1$

16. $3x^2 + 2x + 4xy$

17. $3x^2y - 3xy + 6xy^2$

18. $3y^2 + 9y - 6y^3 + 3x^2y + 6xy^2 + 9xy$

FACTORING COMPLETELY

As you have seen in this lesson, there are often many ways to factor a polynomial. However, there is only one way to factor it *completely.* For example, $(4x + 8)(3x + 9)$ is factored, but to factor it completely you would have to factor 4 out of $(4x + 8)$ and 3 out of $(3x + 9)$.

Factor completely.

19. $(2x + 6)(3x + 6)$

20. $4(x^2 + 5x + 6)$

21. $4x^2 + 40x + 64$

22. $2x^2 + 8x + 8$

23. $3x^2 + 21x + 30$

24. $2x^2 + 26x + 72$

25. $x^3 + 5x^2 + 6x$