LESSON 5.2

Constant Products

MILES PER GALLON

If you plan to take a trip of 100 miles, the amount of gas you need depends on how many miles per gallon your vehicle gets. Some very large recreational vehicles get only about 5 miles per gallon, while a scooter can get 100 miles per gallon.

1. Copy and complete the table to show how many gallons of gasoline you should buy if your vehicle gets the mileage indicated. Continue the table up to 100 miles per gallon.

Mileage (miles per gallon)	Gasoline needed (gallons)	Total trip distance (miles)
5	___	100
10.5	___	100
20	___	100

2. Graph the (x, y) pairs in the first two columns of the table.

3. Describe your graph in words. If you were to extend your graph, would it go through the origin? Would it touch or cross the axes? Explain.

CONNECTING THE DOTS

4. Make a table containing these points and plot the (x, y) pairs on a Cartesian graph.

$(2, 12)$ $(3, 8)$ $(4, 6)$ $(8, 3)$

5. Describe the pattern of the (x, y) pairs in problem 4

a. in words; b. using algebra.

6. a. Find five more (x, y) pairs that fit this pattern and add the points to your table and graph. Use positive values for x. Include some fractional values.

b. Add five more (x, y) pairs to your table and graph. This time use negative values for x, including some fractional values.

7. Study the points on your graph. If necessary, add more points so that you can answer the following questions.

a. Which quadrants do your points lie in? Why?

b. Can you find a point on the y-axis that fits the pattern? Can you find a point on the x-axis? Explain.

c. If you were to connect the points with a smooth curve, would the curve go through the origin? Explain.

8. Add to your graph a point that fits the pattern and

a. has an x-value less than 1/2;

b. has a y-value less than 1/2;

c. has an x-value greater than 24;

d. has a y-value less than -24.

9. Study your answers to problems 4-8. Then very carefully connect the points with a curve. Your curve should have two parts *that are not connected* to one another.

a. Describe the graph.

b. Explain why the two parts are not connected.

10. For (a-d), find several pairs of numbers (x, y) that satisfy the description. Plot these points on your graph.

a. x is positive and xy is more than 24.

b. x is positive and xy is less than 24.

c. x is negative and xy is more than 24.

d. x is negative and xy is less than 24.

11. Plot a point (x, y) such that $xy = 24$ and $x = y$.

We could call the curve you drew in problem 9 a *constant product* graph, since the product of the coordinates of every point is the same number. We could graph many other *constant product* graphs of the form $xy = P$, where P could be any number we choose.

12. Experiment with the graphs of some equations of the form $xy = P$. Try several different positive values for P. Then try several different negative values for P. For each one, make a table of at least eight (x, y) pairs having the same product. Then draw a graph. Draw all your graphs on the same pair of axes.

13. Report Write a report summarizing your findings about constant product graphs. Your report should include neatly labeled graphs with accompanying explanations. Include answers to the following questions:

- What is the shape of the graph?
- Without drawing the graph, could you now predict which quadrants the graph would be in, if you knew the value of P? Explain.
- Do any of the graphs go through the origin? If not, do you think you could find a value of P so that the graph would go through the origin? Explain.
- Where can you find points whose product is not P?
- Comment on anything you notice about the x-intercepts and y-intercepts.
- Do any of your graphs intersect? Explain why or why not.

OTHER GRAPHS

In order to graph some functions, Tomas made tables of values, plotted the points, and connected the dots. (For one of the equations, he tried two different ways.) He asked his teacher if he had done it right. Mr. Stephens answered that the individual points had been plotted correctly, but he asked Tomas to think about how he had connected them. He said, *"Every point on the graph, even the ones obtained by connecting the dots, must satisfy the equation."* Tomas didn't understand. Mr. Stephens added, "Check whether you connected the dots correctly, by substituting a few more values of x into the equation. Use your calculator to see if the y-value you get is on the graph you drew." Tomas still didn't understand.

Name: Tomas

$y = x - 2$

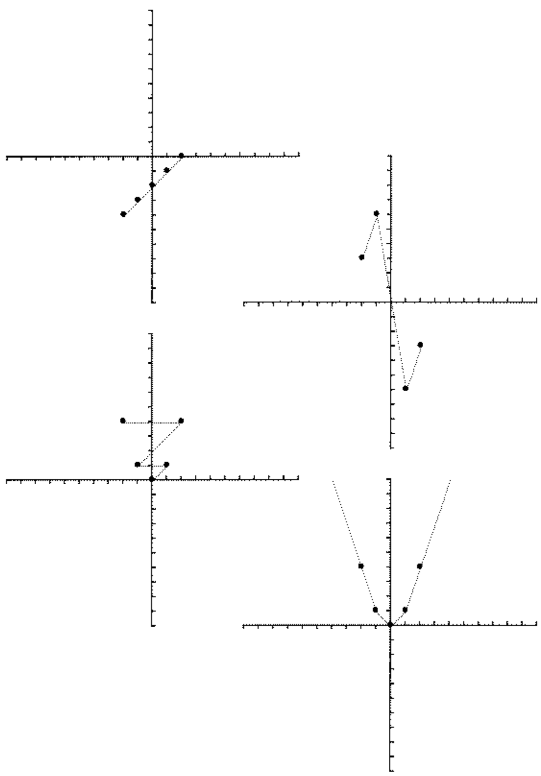
x	y
-2	-4
-1	-3
0	-2
1	-1
2	0

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y = -6/x$

x	y
-2	3
-1	6
0	imp.
1	-6
2	-3



14. Report Explain how Tomas can improve his graphs. Show your calculations. Give Tomas advice he can understand, on:

- how to label axes, points, and graphs;
- how to connect the dots correctly;
- how to extend the graph to the left and right;
- how a calculator can help.