LESSON 4.3

Polynomial Functions

You will need:

graph paper 

Definition: A *polynomial function* is a function of the form $y =$ a polynomial.

1. **Exploration** Which of these polynomial functions do you think have graphs that are straight lines? Which have curved graphs? Explain why you think so.

a. $y = x^2$ b. $y = 2x - 1$

c. $y = 2x^2$ d. $y = x^3$

ORDER OF OPERATIONS

2. Make a table of at least eight (x, y) pairs for each function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table. Label each graph with its equation. You will need to refer to these graphs later.

a. $y = x^2$ b. $y = x^3$

To make a table of values for graphing $y = -x^2$, we have to know what the expression $-x^2$ means. Does it mean *square x, then take its opposite* or *take the opposite of x, then square it*? Which operation should be done first?

To avoid this kind of confusion, mathematicians have agreed on the following rule.

Rule: *Exponentiation should be performed before other operations.* To change this order, we have to use parentheses.

Examples:

- $-x^2$ means *square x, then take the opposite.*
- $(-x)^2$ means *take the opposite of x, then square the result.*

3. Make a table of at least eight (x, y) pairs for each function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table. Label each graph with its equation.

a. $y = (-x)^2$ b. $y = -x^2$

4. Compare your graphs in problem 3 with the graph of $y = x^2$. Explain what you observe.

5. Graph these polynomial functions.

a. $y = -x^3$ b. $y = (-x)^3$

6. Compare your graphs in problem 5 with the graph of $y = x^3$. Explain what you observe.

DEGREE

Definition: The degree of a polynomial function in one variable is the highest power of the variable that appears in the polynomial.

Examples: $y = x^3$ and $y = x^2 + 2x^3$ are both third-degree polynomial functions. The equation $y = 2x$ is first-degree, and the equation $y = 1$ is zero-degree.

7. What is the degree of each of these polynomial functions?

a. $y = 5 + x^2 - x$

b. $y = 4x^3 - 3x^2 + 5$

c. $y = 45$

8. Make a table of at least eight values for each third-degree function. Use negative numbers and fractions as well as positive whole numbers in your tables. Then make a graph from each table.

a. $y = 2x^3$ b. $y = x^3 + 1$

c. $y = -x^3 - 2$

9. Repeat problem 8 for these second-degree functions.

a. $y = x^2 - 1$ b. $y = -3x^2$

c. $y = -x^2 + 2$

10. Graph these first-degree functions.

a. $y = 5x$ b. $y = x$

c. $y = -2x + 1$

11. Graph these zero-degree functions.

a. $y = 4$ b. $y = -3$

c. $y = 0$

THE EFFECT OF DEGREE

12. Tell whether each sentence (a-b) could describe the graph of a zero-degree, first-degree, second-degree, or third-degree polynomial function. More than one answer may be possible for each description.

a. The graph is a straight line.

b. The graph is a curve.

13. Repeat problem 12 for these descriptions.

a. The graph goes through the origin.

b. The graph never crosses the x-axis.

c. The graph never crosses the y-axis.

14. Repeat problem 12 for these descriptions.

a. The graph passes through quadrants I and III only.

b. The graph passes through quadrants II and IV only.

c. The graph passes through quadrants I and II only.

15. Summary How does the degree of the equation affect its graph? Write a summary explaining everything you know about this.

16.

a. Make a table of values and graph the function $y = 24/x$.

b. Is this a polynomial function? Explain.

REVIEW *POSSIBLE OR IMPOSSIBLE?*

17. Decide whether each of the following situations is possible or impossible. If it is possible, give an example. If it is impossible, explain *why* it is impossible. Can you subtract

a. a negative number from a negative number to get a positive number?

b. a negative number from a negative number to get a negative number?

c. a negative number from a positive number to get a positive number?

d. a negative number from a positive number to get a negative number?

e. a positive number from a negative number to get a negative number?

f. a positive number from a negative number to get a positive number?