LESSON 1.9

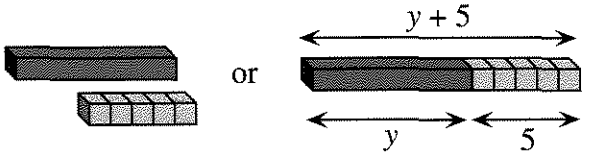
Adding and Multiplying

You will need:

the Lab Gear 

ADDITION

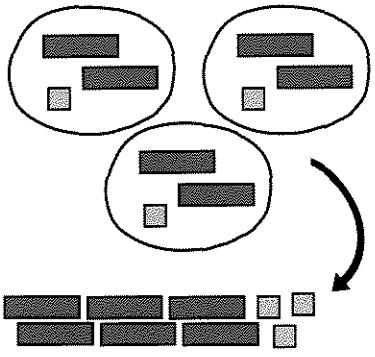
Using the Lab Gear, the addition $y + 5$ can be modeled in two ways. You can show two collections of blocks, y and 5. Or you can line up the blocks to get a figure that has length $y + 5$. Both methods are shown here.



1. Sketch this addition both ways, $3x + 2$.

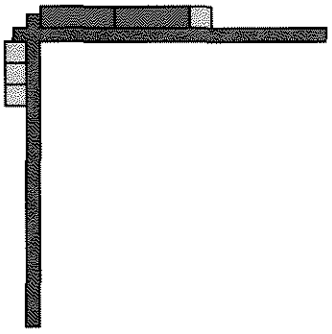
MULTIPLICATION

The multiplication $3 \cdot (2x + 1)$ can be modeled in two ways. One way is to show three collections of $2x + 1$.

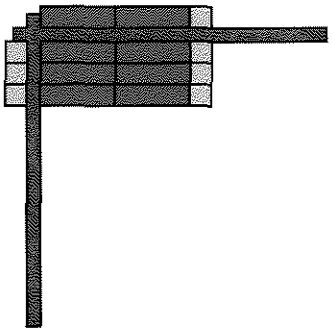


As you can see in the figure, $3 \cdot (2x + 1) = 6x + 3$.

The other way is to use the corner piece. First set up the factors (3 and $2x + 1$) on the outside.



Then make a rectangle having those dimensions.



The rectangle represents the product. Again you see that $3 \cdot (2x + 1) = 6x + 3$. This is the familiar *length* · *width* = *area* formula for a rectangle.

2. Sketch this multiplication two ways, $2 \cdot (x + 3)$.

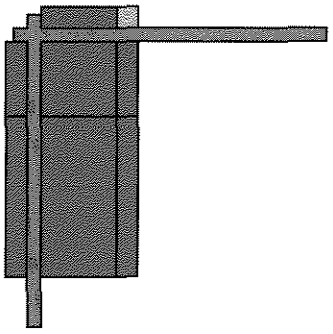
a. Use collections of blocks.

b. Use the corner piece.

3. What were the length, width, and area of the rectangle in problem 2?

With any factors of degree 0 or 1, you can model the multiplication in the corner piece.

4. What multiplication is shown in this figure?



5. Multiplying the x by the x gave x^2. What other multiplications do you see in the figure above?

6. Multiply with the corner piece.

a. $3x \cdot 2$ b. $3 \cdot 2x$

c. $2x \cdot 3$ d. $2x \cdot 3y$

7. Multiply with the corner piece.

a. $5(x + 1)$ b. $x(x + 3)$

8. Find the area of a rectangle having the sides given below. For each write an equation of the form *length times width = area.*

a. 5 and $x + 3$

b. x and $2x + 5$

9. Find the sides of a rectangle having the given area. Each problem has at least two solutions. Find as many of them as you can and write an equation for each.

a. $6x$ b. $6x^2 + 3x$

10. These equations are of the form *length times width = area.* Use the Lab Gear to help you fill in the blanks.

a. $y \cdot$ _____ $= y^2 + xy$

b. $(x + 2) \cdot$ _____ $= 3x + 6$

c. (_____ $+ 3) \cdot x = 2xy + 3x$

Understanding the area model of multiplication will help you avoid many common algebra errors.

ORDER OF OPERATIONS

The figure above showed a multiplication. Some students write it like this: $x + 1 \cdot x + y$. Unfortunately, someone else might read it as *add the three terms*: x, $1 \cdot x$, and y. Simplified, this would be $x + x + y$, or $2x + y$. But the intended meaning was equivalent to $x^2 + xy + x + y$, as you can see on the figure. To avoid this kind of confusion, mathematicians have agreed on the following rule.

Rule: When the operations of multiplication and addition (or subtraction) appear in the same expression, *multiplication should be performed first.* If we want to change this order, we have to use parentheses.

This means that one correct way to write the multiplication in the figure is $(x + 1)(x + y)$, which can mean only *multiply $x + 1$ by $x + y$.*

11. a. Show $2 \cdot x + 5$ with the Lab Gear. Sketch.

b. Next to your sketch show $2 \cdot (x + 5)$ with the Lab Gear. Sketch it. Keep the blocks on the table for the next problem.

12. a. Copy both collections of blocks from problem 11, substituting 1 for x. What is each expression equal to?

b. Repeat, using 5 for x.

c. Repeat, using 0 for x.

13. Can you find a value of x for which $2 \cdot x + 5 = 2 \cdot (x + 5)$? If so, what is the value? If not, why can't you find a value?

14. **Exploration** Insert parentheses in each expression, so as to get many different values. What are the greatest and smallest values you can find for each one?

a. $0 \cdot 1 + 2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + 8 \cdot 9$

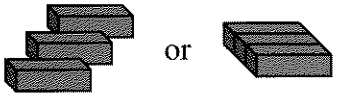
b. $0 + 1 \cdot 2 + 3 \cdot 4 + 5 \cdot 6 + 7 \cdot 8 + 9$

THE SAME OR DIFFERENT?

Students sometimes confuse $3 + x$ with $3x$. With the Lab Gear, it is easy to see the difference. $3 + x$ involves addition.

or

$3x$ involves multiplication.



15. Find the value of $3 + x$ when:

a. $x = 0$ b. $x = 5$

c. $x = 0.5$

16. Find the value of $3x$ when :

a. $x = 0$ b. $x = 5$

c. $x = 0.5$

17. For most values of x, $3x$ does not equal $3 + x$. In fact there is only one number you can substitute for x that will make $3 + x$ equal to $3x$. Use trial and error to find this number.

18. Build these expressions with the Lab Gear. Sketch. Which two are the same?

a. $6xy$ b. $2x + 3y$

c. $2x \cdot 3y$ d. $5xy$

19. Build and sketch these two expressions with the Lab Gear.

a. $2x + 3y$ b. $2xy + 3$

20. Use trial and error to find a pair of values of x and y that will make the two expressions in problem 19 have the same value.

21. Use the Lab Gear to show each expression. Sketch.

a. $5 + x + y$ b. $5 + xy$

c. $5x + y$ d. $5xy$

22. Choose values for x and y so that all four expressions in problem 21 have different values.