Teacher Notes on the Quadrilaterals Unit

Students make conjectures about quadrilaterals, diagram them, and then write proofs of these or find counterexamples. Proofs can use previously established results, modeling the way new mathematical knowledge comes into being. Keeping track of what has been proved to date is important, and not so easy. A key tool for this, a few days into the unity, is the "map of quadrilateral theorems." That map should gradually replace the original list of conjectures as a source of ideas.

Writing proofs is the main content of the unit, even though there are no pages about it, since the choice of conjectures to prove depends on your class.

Actually, that is putting it too strongly: you can influence the sequencing of what to prove, by alternating your suggestions with student suggestions, by steering students away from theorems they are not ready to prove, and so on. The "List of Quadrilateral Theorems" in these Teacher Notes can be helpful in working out a good sequence.

Interesting conversation to have: "Proving x would be easier if we had already proved y. Let's see try to prove y first."

This approach is unique to our program, and tries to make proof *as a method to establish truth* meaningful to the majority of students —instead of proof as something you do because that's what the teacher inexplicably insists on. The key message is that proofs and counter-examples are the tools we use to figure out whether conjectures are true.

It is not of primary importance that students get out of this unit knowing any particular theorems. The focus is on understanding what proof is about, and on getting better at proving and disproving.

Quadrilateral Conjectures

This activity is the foundation of the whole unit. The unit models the way mathematicians work. They start with conjectures, which they prove or disprove, using previously proved results.

Before starting the individual and group work, make sure they know what all the words mean. (consecutive, bisect each other, etc.) -- for example, "opposite angles are equal" means that *both* pairs of opposite angles are equal.

Make sure a student ends up with a copy of their group's conjectures, and e-mails it to you. Combine them all into one organized file, and give it to the students the next day (see the **Teacher Notes** file for an example – the conjectures are grouped into sections, and within each section they are put in alphabetical order to lose any sense of who came up with what, and make it about the ideas, without any individuals or groups "owning" the conjectures.)

Over time, students will prove or disprove many of the conjectures. They may also add conjectures to the list. This work should happen mostly in class, though you may choose specific conjectures to prove or disprove as homework.

Keep an ongoing class list of what has been proved. (Use the map on the next page for this.)

Sample List of Quadrilateral Conjectures

[This list came out of Henri's class in 04-05. If you're lucky, your list will be shorter!]

Obviously True?

If all angles are equal, then opposite angles are equal.
If all angles are equal, then opposite sides are equal.
If all diagonals are equal, perpendicular, bisectors, then all angles are equal.
If all diagonals are equal, perpendicular, bisectors, then all sides are equal.
If all sides are equal, then all angles are equal.
If all sides are equal, then are opposite sides are equal.
If all sides are equal, then one pair of sides are both parallel and equal.
If equal diagonals bisect each other then all sides are equal.
If one pair of sides are both parallel and equal, then opposite sides are parallel.
If opposite sides are equal, then opposite angles are equal.
If opposite sides are equal, then opposite angles are equal.
If opposite sides are parallel and equal, then diagonals are equal.
If the diagonals are equal, then at least one pair of opposite sides are parallel.
If the opposite sides are parallel, the diagonals are equal.

Probably True?

If 2 pairs of consecutive sides are equal, the diagonals are perpendicular.
If 2 pairs of opposite angles are equal, then consecutive angles add up to 180.
If all angles are equal, then the diagonals are equal.
If all sides are equal, opposite angles are equal.
If all sides are equal, then all angles are equal, then the diagonals will be perpendicular.
If all sides are equal, then the diagonals are equal.
If consecutive angles are supplementary, then all angles are equal.
If one pair of sides are both parallel and equal then two pairs of consecutive sides are equal.
If one pair of sides are both parallel and equal, then opposite sides are equal.
If one pair of sides is parallel and equal then all angles must be equal.
If opposite angles are equal then opposite sides are equal.
If opposite are parallel, then the diagonals bisect each other.
If opposite sides are equal, then consecutive angles add up to 180 degrees.
If opposite sides are equal, then consecutive angles add up to 180o.
If opposite sides are parallel, then one pair of sides are parallel and equal.
If the diagonals are equal, then the sides must be equal.
If two pairs of consecutive angles are equal, then the diagonals are perpendicular.
If two pairs of consecutive sides are equal, then opposite angles are equal.
If two sides are parallel but unequal, and the other two are equal but not parallel, then the diagonals are equal.
If two sides are parallel but unequal, and the other two are equal but not parallel, then the diagonals are equal.

Probably False?

If all angles are equal, then the diagonals are perpendicular.
If all opposite sides are equal then all sides are equal.
If all sides are equal, then all angles are equal.
If consecutive angles add up to 180 degrees, then all angles are equal.
If consecutive angles add up to 180, then opposite sides are equal.
If opposite angles are equal, then all angles are equal.
If opposite angles are equal, then all sides are equal.
If opposite sides are equal then all sides are equal.
If opposite sides are equal, then opposite sides are parallel.
If two pairs of consecutive angles are equal, the diagonals bisect each other.

Obviously False?

If all angles are equal then all sides are equal.
If all angles are equal, then all sides are equal.
If consecutive angles add up to 180 degrees, then all angles are equal.
If opposite sides are equal, then all sides are equal.
If opposite sides are equal, then opposite sides are parallel.
If opposite sides are equal, then opposite sides are parallel.
If opposite sides are parallel, then opposite sides are equal.
If opposite sides are parallel, then opposite sides are equal.
If two pairs of consecutive angles are equal, then one pair of sides are both parallel and equal.
If two sides are parallel, but unequal, and the other two are equal, but not parallel, then all angles are equal.
In a trapezoid, one pair of opposite sides, are parallel but unequal, then the other 2 sides are equal but not parallel.

Map of Theorems about Quadrilaterals

The idea is to enter things like “opposite sides are equal” and connect them to the definitions with arrows, once the results are proved. Example: “opposite sides are equal” —> “parallelogram” —> “opposite sides are equal”. One can also connect the definitions with arrows, once those results are proved. Example: “rhombus” —> “parallelogram”.

This sheet helps the transition from the rather abstract format of the Making Quadrilateral Conjectures sheet to the usual way to discuss these theorems. It also provides a way to keep track of what has been proved.

See the next page for a sample (“target”) map with arrows drawn in. To draw the arrows, use Word’s “Drawing” toolbar. Or use the SmartBoard file.

You can keep adding to it in your copy, which you project on the screen, and students can update their copy by hand. Towards the end of the unit, ask them to write a list of the theorems.

Do not show students the target sheet: it is strictly for your reference.

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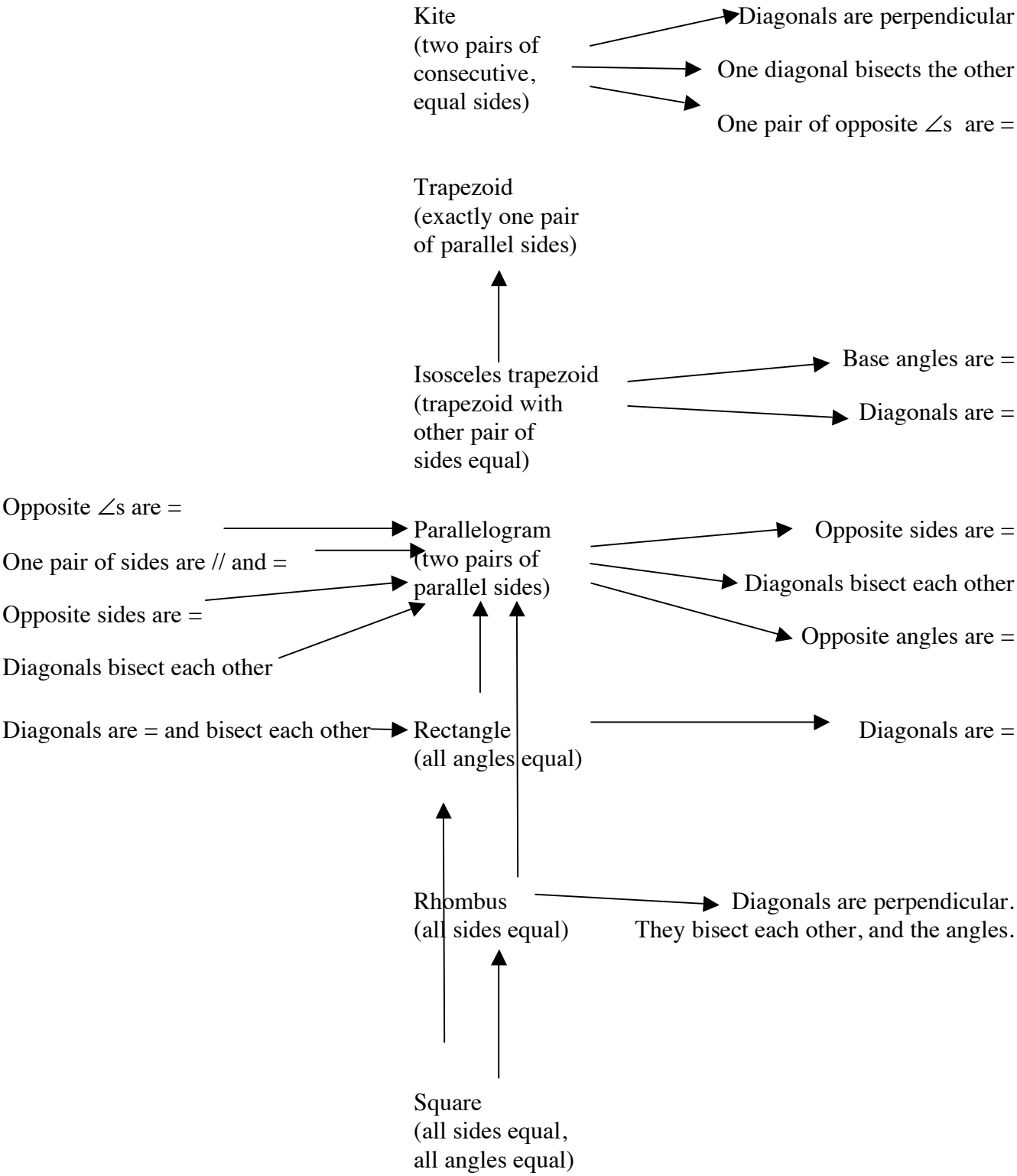
The target map is based on the theorems in Chakerian, chapter 4, plus a few additional results from Henri’s class in 02-03. It is realistic to get most of these theorems, but it’s not crucial to get every single one.

Do not show the target map to your students! Each class needs to make its own map.

Additional results are possible. For example, students may prove something is true for a rectangle, which is actually true in a more general case of a parallelogram. Accept such results, and save for later a discussion of how to economically show them on the map: they only need to be shown for the parallelogram, and one can use transitivity to see that they are also true for the rectangle.

In fact, this map provides a good context for discussion of transitivity — preferably towards the end of the unit. (Transitivity: if a implies b and b implies c, then a implies c.)

Map of Theorems About Quadrilaterals [Target]



List of Quadrilateral Theorems

The list below is for the teacher to use in choosing what theorems to prove. Do not share this with the students, as that would sabotage the basic idea of the unit, which is for students to figure out what to prove, and prove it, instead of having it all mapped out by the teacher. Think of this sheet as a backup.

The theorems are given here in the order they come up in Chakerian's *Geometry: A Guided Inquiry*. Use the list to help weaker students choose easier theorems to prove (earlier in the list is easier.) Use it also to choose theorems for everyone to prove, as it makes it easier for students to compare their answers, or theorems to prove at the board when needed for a class discussion, or theorems to prove on quizzes or tests.

However, this should not prevent students from selecting a different path through these theorems, or proving results not on this list. Make sure you keep track of what they prove, and check it off this list as you go.

Opposite sides of a parallelogram are equal.

Opposite angles of a parallelogram are equal.

The diagonals of a parallelogram bisect each other.

If opposite sides of a quadrilateral are equal, it is a parallelogram.

If the diagonals of a quadrilateral bisect each other, it is a parallelogram.

The diagonals of a rectangle are equal.

If the diagonals of a parallelogram are equal, it is a rectangle.
(note that if the diagonals of a general quadrilateral are equal, it is not necessarily a rectangle!)

If two sides of a quadrilateral are both parallel and equal, it is a parallelogram.

There may be theorems about kites and trapezoids in Jacobs' *Geometry*. Those belong at the bottom of this list.