Parabola Similarity

In this activity, we will use GeoGebra in Algebra mode to explore parabola similarity.

Basics

First, let us make sure we have some needed GeoGebra basics.

1. Graph a parabola by typing in the Input Bar at the bottom of the screen:
   f(x) = x^2 + 2x +3 (or use your own parameters, or even sliders for a, b, c)
   Find the vertex, using only geometric tools.

2. Graph another parabola. Find the vertex using the **Min** command if it is a "smile" parabola. (If it is a "frown" parabola, use the **Max** command, which works the same way.)
   V=**Min**[<Function>, <Start x-Value>, <End x-Value>]
   Replace the place holders including the < and >, as indicated.

3. In a new window, show the grid. Make a vector with endpoints at lattice points (where grid lines intersect.) Translate a parabola, using this vector.

4. Make a new point. Dilate a parabola using this point as center. Choose your scale factor so that the image shows up well in the window.

All Parabolas Are Similar

5. In a new window, graph two parabolas whose equations have different *a* coefficients. Use zero, one, or two isometries followed by a dilation to show that the two parabolas are similar.

6. **Challenge:** How is the scaling factor related to the equations of the two original parabolas?

7. **Challenge:** Find a dilation that will take your first parabola to your second parabola without a need for any isometries.

The Scaling Factor

8. Graph $y = x^2$ in a new window. Make a point at the origin; call it O. One way to do this is to type O=(0,0) in the Input Bar. Make a slider; call it s.

9. Make a point A on the graph. Dilate A with center O and scaling factor s. The resulting point is A'.

10. Explain the following statements:
    a. If the coordinates of A are (x_A, y_A), we have $y_A = x_A^2$.
    b. If the coordinates of A' are (x,y), we have $x = sx_A$ and $y = sy_A$.

11. Use these facts and some algebra to write y as a function of x and s.

12. Graph the function. Verify that as you move A on the original graph, A' moves on the new graph.

13. **Conclusion:** The graph of $y = ax^2$ is a dilation of the graph of $y = x^2$. Where is the center of dilation? What is the scaling factor?

For more information about the geometry of the parabola, including parabola similarity, see:
http://www.mathedpage.org/parabolas/geometry/