Bisector Theorems

Make two conjectures:

1. The set of points that are equidistant from the endpoints of a line segment is the ____________________ ___________________ of the line segment.

2. By definition, the distance from a point to a line (or ray, segment) is always the __________________ distance!

3. The set of points that are equidistant from the rays of an angle is the ______________ _________________.

4. Write each of the conjectures in if-then form, accompanied by an if-then diagram.

5. Write the converse of each conjecture, accompanied by an if-then diagram.

6. Prove four theorems.

Basic Constructions

For each construction, use compass, straightedge, and/or patty paper.
Take some notes on how you did it.

1. All points equidistant from a given point

2. All points equidistant from the endpoints of a segment

3. The distance from a point to a line

4. All points equidistant from the sides of an angle

5. An equilateral triangle

6. An isosceles triangle that is not equilateral

7. Bonus: A rhombus

8. Bonus: A square

9. Bonus: Some points equidistant from a point and a line

Circles Through Points

You may use interactive geometry software from now on. (If using GeoGebra, stick to the Basic Geometry tools.)

For each problem, start by drawing the given, then do the construction. *The construction is not correct if you can ruin it by moving the given.*

At each checkpoint (√), show your teacher your figure, and save it. You can call the files **cp3**, **cp4**, **tc3**, etc, or use names that are more meaningful.

1 Given a segment, construct a circle that has the segment as its diameter.

2 Given two points, construct three different circles that go through both. **Hint**: first find where the centers should be located.

3 Given three points, construct a circle that goes through all three. **Hint**: use what you found out in problem 2. √

4 Given a rectangle, construct a circle through its vertices. √

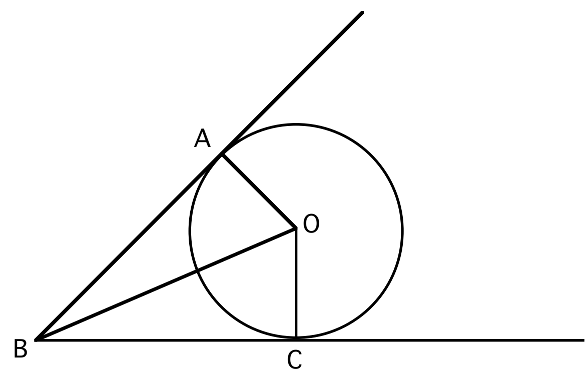
Tangent Circles

For each problem, start by drawing the given, then do the construction. *The construction is not correct if you can ruin it by moving the given.*

At each checkpoint (√), show your teacher your figure, and save it. You can call the files **cp3**, **cp4**, **tc3**, etc, or use names that are more meaningful.

1 Given a line and a point not on it, construct a circle centered at the point and tangent to the line.

2 Given two lines, construct three different circles tangent to both. **Hint**: first look at the triangles in this figure.



3 Given a triangle, construct a circle tangent to all three sides. **Hint**: use what you learned in problem 2. √

4 Given a triangle, construct a circle tangent to one side, and to the extensions of the other two, on the outside of the triangle. √

(Optional: given a triangle, construct four circles tangents to the three sides — one inside, and one outside on each side.) √

5 Given a line and a point not on it, construct a circle tangent to the line, which passes through the point.

6 Find three more circles that satisfy the conditions of problem 5. (Hint: for each one, start with the point of tangency.) √

7 Given a circle and a point not on it, construct a circle through the point, tangent to the circle.

8 Find three more circles that satisfy the conditions of problem 7. √

Tangent Lines

Important: All these constructions depend on knowing that a line tangent to a circle is ____________ to the ________ at the point of contact.

1. Given a circle and a point on it, construct a tangent to the circle through the point.

2. Given a circle and a line, construct a tangent to the circle that is parallel to the line.

3. Given a circle and a line, construct a tangent to the circle that is perpendicular to the line.

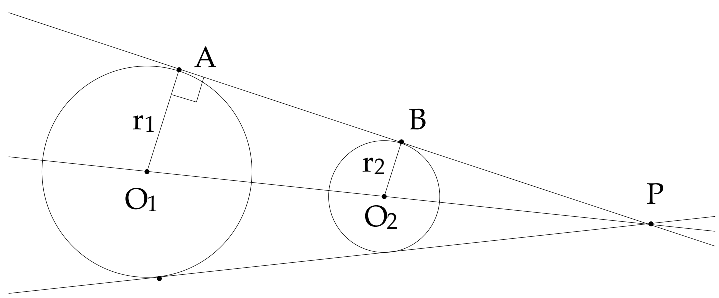
Before the next tangent line construction, you need to solve this problem:

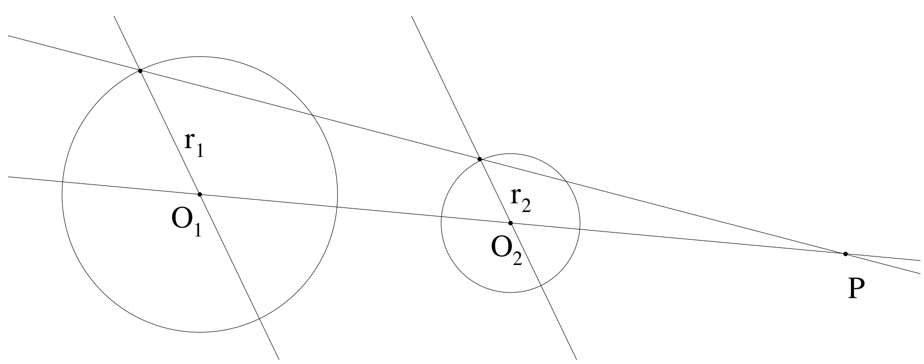
4. Given a line segment, construct three right triangles that have the segment as their hypotenuse. (Hint: use something you know about inscribed angles.)

5. Complete the sentence, using what you learned in #4: Given two points A and B, the location of all the points P such that $\angle$ APB = 90° is __________________________

6. Given a circle and a point outside it, draw both tangents to the circle through the point.

Challenge! The next two constructions are tricky. Study these figures to help you with #7. (Hint: Think of similar triangles, and show that $PO_1 / PO_2 = r_1 / r_2$ in both figures.)





7. Given two non-congruent circles, draw a line tangent to both, on the outside.

8. Given two non-congruent disjoint circles, draw a line tangent to both, on the inside.

Lines in a Triangle 1

On this page, use compass, straightedge, and/or patty paper.

Theorem: The perpendicular bisectors of the sides of a triangle meet in one point, the center of the circumscribed circle.

(The *circumscribed circle* is the circle through the vertices of a triangle.)

1. Proof: write the proof of the theorem, based on your teacher's explanations.
2. Using a straightedge, draw a large triangle on patty paper. Label the vertices A, B, C. Make the perpendicular bisectors of all three sides by folding. Call their meeting point O.
3. Using a compass, draw the circumscribed circle.

Theorem: The angle bisectors of the angles of a triangle meet in one point, the center of the inscribed circle.

(The *inscribed circle* is the circle tangent to the three sides of a triangle.)

4. Proof: write the proof of the theorem. It is not unlike the proof above.
5. Using a straightedge, draw a large triangle on patty paper. Label the vertices A, B, C. Make the angle bisectors of all three angles by folding. Call their meeting point P.
6. Drop a perpendicular by folding, from P to one of the sides of the triangle.
7. Using a compass, draw the inscribed circle.

Lines in a Triangle 2

On this page, use interactive geometry software.

Definition: In a triangle, the line connecting a vertex to the midpoint of the opposite side is called a *median*.

1. Draw a triangle ABC. Let D and E be the midpoints of AC and BC respectively. Draw the medians AE and BD. Call the point where they meet M. Let F and G be the midpoints of BM and AM respectively.

2. Draw the quadrilateral DEFG.
   a. Prove it is a parallelogram. (Hint: how is DE related to AB? How is GF related to AB?)
   b. Prove that M is 1/3 of the way down the median from the side. (Hint: the diagonals of a parallelogram bisect each other.)
   c. Prove that the third median must pass through M. (Hint: use what you proved in #2b.)

Theorem: The medians of a triangle meet in a single point, called the *centroid* or center of gravity of the triangle.

Definition: In a triangle, the perpendicular to a side through the opposite vertex is called the *altitude*.

3. Draw a triangle ABC. Construct parallels to the three sides, through the opposite vertices, creating a new, larger triangle DEF.

4. Prove that the altitudes of Δ ABC meet in a single point. Hint: the altitudes of ΔABC are what in ΔDEF?

Theorem: The altitudes of a triangle meet in a single point, called the *orthocenter*.

The following result is difficult to prove, but is fun to discover.

5. Draw a triangle ABC. Construct all four centers, but hide all the construction lines. Which three centers are *always* collinear?