1. Rectangular Pens: Constant Perimeter

You will need: graph paper, a graphing calculator.

1. **Exploration.** You want to make a rectangular pen for Stripe, your pet zebra. Even though Stripe takes many walks around town, you want to make sure she has as much space as possible inside the pen. You have 50 feet of fencing available. If you use all of it to make the pen, what is the biggest area possible? Find out by trying various dimensions for the pen.

Width as a Function of Length

You have 28 feet of fencing to make a rectangular pen. There are many possible dimensions for this pen. One possible pen would be 10 feet wide by 4 feet long. In this section, you will investigate how the length and width change in relation to one another if you keep the perimeter constant.

2. a. On graph paper, draw axes, and at least six pens with a perimeter of 28.
   b. The upper right hand corner of the pen on the figure has been marked with a • and labeled with its coordinates. Do this for the pens you drew. Then connect all the points marked with •. Describe the resulting graph.

3. a. Make a table showing all the coordinates on your graph. Look for a pattern and make three more entries in the table.
   b. Write an equation for the function described by your graph and table.

4. The point whose coordinates are (4,10) is on the graph.
   a. What does the sum of these numbers represent in this problem?
   b. What does the product represent?

5.
   a. What is the greatest possible length of a pen? How can you see this on your graph?
   b. How many rectangles are possible if the dimensions are whole numbers? How many are possible otherwise?
   c. Explain why the graph should not be extended into quadrants 2 and 4.

6. If you increase the length by 1 foot, does the width increase or decrease? Does it change by the same amount each time? Explain.

Area as a Function of Length

In the previous section, you may have noticed that the area of the rectangles changed even though the perimeter remained constant. In this section, you will investigate how the area changes as a function of length, if you keep the perimeter constant.

7. Write the area of the corresponding rectangle next to each of the points marked with a • on the graph from Problem 2.

8. Make a graph of area as a function of length. Show length on the x-axis and area on the y-axis. Connect the points on your graph with a smooth curve. What kind of curve is it?

9.
   a. Label the highest point on your graph with its coordinates. Interpret these two numbers in terms of this problem.
   b. Where does the graph cross the x-axis? What do these numbers mean?
   c. If you increase the length by 1 foot, does the area increase or decrease? Does it change by the same amount each time? Explain.

10. **Summary.**
    a. Describe in words how you would find the area of the rectangular pen with perimeter 28 if you knew its length.
    b. If the perimeter of a rectangular pen is 28 and its length is L, write an algebraic expression for its area in terms of L.
    c. If you had 28 feet of fencing and wanted to make the largest possible rectangular pen, what would its length, width, and area be? Explain.

11. **Generalization.** Say the perimeter of a rectangle is P and its length is L. Write the following expressions in terms of P and L. (A sketch may help.)
    a. An expression for the width.
    b. An expression for the area.

12. **Generalization.** Explain how to find the length that gives the maximum area. Write an algebraic expression for it in terms of P only.

2. Advanced Penmanship

You will need: graph paper.

Pen Partitions

Assume that you have 50 feet of fencing to build a rectangular pen. You plan to use the garage wall as one side of the pen, which means you only need to use your fencing for three of the four sides. Since you are considering adopting more pets, you want to investigate what happens when you use some of the fencing to divide the pen into two or more parts by building partitions inside the pen, at a right angle to the wall.

1. Make a rough sketch of what this pen might look like,
   a. with no internal partitions
   b. divided into two sections

2. With no partitions, is it possible to get a square pen? If so, what are its dimensions?

3. With one partition is it possible to get two square sections? If so, what are their dimensions?

Call the side of the pen parallel to the wall the length, and the distance between the wall and the side opposite the wall x.

4. Imagine you are dividing the pen into two parts. Make a table with three columns: x, the length, and the total area of the pen.

5. Look for patterns in your table. Express algebraically as functions of x:
   a. the length b. the area

6. What is the equation that expresses the width as a function of x, if the pen is divided into the given number of parts. (Make sketches. If you need to, make tables as in Problem 4.)
   a. 1 b. 3
   c. 4 d. * n

7. Repeat Problem 6, but this time find the area as a function of x.

Maximizing Area

8. If you have to use part of the 50 feet of fencing for a partition to divide the pen into two equal parts, what is the largest total area you can get for the enclosure? Explain how you got your answer, including a graph if necessary.

9. Solve the above problem if you want to divide the pen into 3 equal parts.

10. * Solve the above problem if you want to divide the pen into n equal parts.

11. Look at your solutions for problems 8, 9, and 10. In each case, look at the shapes of the subdivisions of the pen with the largest area. Are they always squares? Are they ever squares? Does the answer to this depend on the value of n? Explain.

12. Look at your solutions for problems 8, 9, and 10. In each case, look at how much of the fencing was used to construct the side parallel to the garage for the pen of maximum area. What fraction of the fencing was used to construct the side parallel to the garage? Does the answer depend on the value of n? Explain.

13. **Discussion.**
    a. Explain how to maximize the area of pens for a given amount of fencing.
    b. Comment on the shape of the pens.
    c. Comment on the fraction of the fencing used for the side parallel to the garage.

14. **Extension:** What is the biggest rectangular area you can enclose if you have 60 feet of fencing, and:
    a. a 10-foot wall
    b. a 25-foot wall
    c. a 40-foot wall