LESSON 11.3

Stairs and Squares

You will need:
- graph paper
- geoboard

STAIR SAFETY

In most houses, stairs have a riser (or rise) of eight inches and a tread (or run) of nine inches. However, safety experts claim that such stairs are the cause of many accidents. They recommend what they call 7/11 stairs: a riser of seven inches, and a tread of eleven inches.

1. What are the slopes of the stairs described in the previous paragraph? (Express the answer as a decimal.)

2. If a staircase makes a vertical rise of about nine feet from one floor to the next, how much horizontal distance does it take
   a. for 8/9 stairs?
   b. for 7/11 stairs?

3. Why do you think 8/9 stairs are more common?

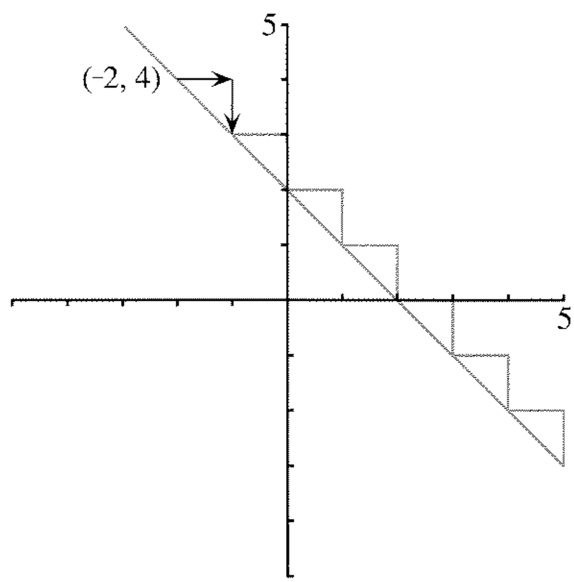
4. **Exploration** Donna wants to build a staircase that is less steep than an 8/9 staircase would be, but that does not take up as much horizontal space as a 7/11 staircase would. What are the possibilities for the riser and tread of Donna's stairs? Make the following assumptions:
- The riser and tread must each be a whole number of inches.
- The riser should be between six and nine inches, inclusive.
- The tread should be between eight and twelve inches, inclusive.

Express your answer numerically and graphically.

STAIRS ON LINES

To build a staircase on the graph of a line:
   a. Sketch the graph.
   b. Find the coordinates of a point on the graph. Call this point the starting point.
   c. Find two numbers (the rise and the run) such that if you draw a step having those dimensions, you end up on the line.

The figure shows a staircase for the line $y = -x + 2$.



The starting point is (-2, 4), the rise is -1, and the run is 1.

5. Create a staircase for the same line, using a different starting point and a different rise and run.

6. Create *two* staircases for each line. They must have a different starting point and a different rise and run.
   a. $y = -4 + 3x$
   b. $y = -0.5x$
   c. $y = 9$

d. $y = \frac{6x - 7}{8}$

e. $y = -2(x - 3)$

7. Find a rise and a run for a staircase connecting the following pairs of points:

a. (3, -5) and (2, 2.5)

b. (-3, 5) and (2, 2.5)

8. A staircase having the given rise and run starts at the given point. What is the equation of the corresponding line?

a. rise = 4, run = 6, point = (-3, 6)

b. rise = -2, run = -3, point = (0, 8)

LATTICE POINTS AND FRACTIONS

Definition: A *lattice point* is a point on the Cartesian plane having integer coordinates.

Examples: (2, 3) is a lattice point, but (4.5, 6) is not.

9. The graph of each of the following equations is a line through the origin. Find two other lattice points on each line.

a. $y = 7x$ b. $y = \frac{2}{3}x$

c. $y = 4.5x$ d. $y = 6.78x$

If a line passes through the origin and the lattice point (9, 8), it will also pass through the lattice points ($9n$, $8n$) for all integer values of n.

10. If a line passes through the origin and the point (2.4, 3.6),

a. what are the lattice points on the line that are closest to the origin?

b. what is a general description of all the lattice points on the line?

c. what is the equation of the line?

11. Do all lines through the origin pass through another lattice point sooner or later? Discuss.

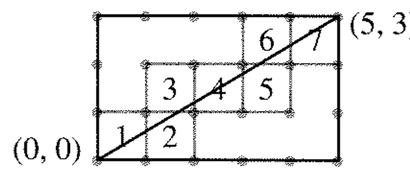
Generalizations

12. What is the slope of a line that passes through the origin and a lattice point (p, q), where $p \neq 0$?

13. Describe the lattice points on the line $y = (p/q)x$, where $p \neq 0$.

GEOBOARD DIAGONALS

If you connect (0, 0) to (5, 3) with a straight line, you go through seven unit squares.



14. Exploration If you connect (0, 0) to (p, q) with a straight line, how many unit squares do you go through? Experiment and look for patterns. (Assume p and q are positive whole numbers.) Keep a record of your work.

Definition: A *lattice line* is a line having equation $x = b$ or $y = b$, where b is an integer.

The following problems are about the diagonal connecting (0, 0) to (p, q). Give answers in terms of p and q.

15. a. How many horizontal lattice lines does it cross? (Look at some specific cases and make a generalization. Do not guess.)

b. How many vertical lattice lines does it cross?

16. How many lattice points does it cross,

a. if the greatest common factor of p and q is 1?

b. if the greatest common factor of p and q is n, where $n > 1$? (Experiment and reason. Do not guess.)

17. The diagonal starts in the first unit square, then every time it crosses a lattice line it enters a new square.

a. If it crosses no lattice points, how many squares does it go through altogether?

b. If it crosses n lattice points, how many squares does it cross?

18. Report How many squares do the diagonals of geoboard rectangles go through? Write an illustrated report, including examples.

DISCOVERY *SLOPE RELATIONSHIPS*

Lines
parallel
perpendicular
symmetric across horizontal line
symmetric across the line $y = x$
symmetric across vertical line

Slopes
opposite
opposite of reciprocal
reciprocal
reciprocal of opposite
same

The first column shows possible relationships between two lines. The second column shows possible relationships between the slopes of two lines.

19. Project Experiment to find out if it is possible to match relationships in the first column with relationships in the second column. (For example, parallel lines have the same slope.) Support your answers with examples, sketches, and explanations.