The Substitution Rule

In the following equations, there are two placeholders, a diamond and a triangle. The **substitution rule** is that within one expression or equation, the same number is placed in all the diamonds, and the same number is placed in all the triangles. (The number in the diamonds may or may not equal the number in the triangles.)

For example, in the equation:

$$\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$$

if you place 2 in the ◊ and 3 in the Δ, you get:

$$2 + 2 + 2 + 3 = 3 + 3$$

Note that even though the diamond and triangle were replaced in accordance to the rule, the resulting equation is not true.

1. **Exploration**: The expression
   $\Diamond + \Diamond + \Diamond + \Delta = \Delta + \Delta$
   is not true with 2 in the ◊ and 3 in the Δ. Find as many pairs of numbers as possible that can be put in the ◊ and Δ to make the expression true. For example, 0 in both the Δ and ◊ make it true. Arrange your answers in a table like this:

◊	Δ
0	0
…	…

   Describe any pattern you notice. Explain why the pattern holds.

For the following equations, experiment with various numbers for ◊ and Δ. (Remember the substitution rule.) For each equation, try to give three examples of values that make it true. If you can only give one, or none, explain why.

2. $\Diamond + \Diamond + \Diamond = 3 \cdot \Diamond$
3. $\Diamond + \Diamond + \Diamond = 4 \cdot \Diamond$
4. $\Diamond + \Diamond + \Diamond = 3 \cdot \Delta$
5. $\Diamond + \Diamond + 2 = 3 \cdot \Diamond$
6. $\Diamond + \Diamond + 2 = 2 \cdot \Diamond$
7. $\Diamond \cdot \Delta = \Delta \cdot \Diamond$
8. $\Diamond \cdot \Delta = \Delta + \Diamond$
9. $\Diamond \cdot \Diamond \cdot \Diamond = 3 \cdot \Diamond$
10. $\Diamond \cdot \Diamond \cdot \Delta = \Diamond + \Diamond + \Delta$
11. Make up a ◊ and Δ equation. Have a classmate answer the question above about it.
12. **Report:** Say that ◊ is x and Δ is y. For each equation above, show both sides with a sketch of Lab Gear blocks. In some cases, the sketches may help you explain whether the equations are always true or not. Write an illustrated report about this.

Recognizing Identities

Definition: An *identity* is an equation that is true for all values of the variables.

Which of these equations are identities? Explain your answers.

1. $3 (x + 2) = 15$
2. $3 (x + 2) = 3x + 6$
3. $4 (2x + 1) = 4 (x + 5)$
4. $4 (2x – 1) = 4 (x – 1)$
5. $4 (2x – 1) = 4 (2x + 1)$
6. $2 (2x + 2) = 4 (x + 1)$
7. $4 (2x – 2) = 2 (4x – 4)$

Using Graphs and Tables

8. Make a table of (x,y) pairs and graph each linear function.
   a. $y = -2 (x – 1) + 2$
   b. $y = -2x + 4$
9. By simplifying the left-hand side, show that $-2(x – 1) + 2 = -2x + 4$ is an identity.
10. For each pair of functions, decide whether both members of the pair would have the same graph. Explain.
    a. $y = 3 – 4x$ $y = 4x – 3$
    b. $y = -6 – 8x$ $y = 8x – 6$
    c. $y = 2x^2$ $y = 2x(x+2) – 4x$
    d. $y = 5 – x$ $y = -x – 5$
    e. $y = -x + 5$ $y = 5 – x$
11. Look at your answers to the previous problem. For each pair that would not have the same graph, graph both functions on the same axes. Find the point where the two graphs intersect and label it on the graph.
12. Which of the pairs of graphs that you drew in the previous problem do not have a point of intersection? Can you explain why this is so?
13. When graphing two linear functions, there are three possibilities: you may get the same line, two parallel lines, or two lines that intersect. Explain what the tables of (x,y) values look like in each case.

Always, Sometimes, Never

While an identity is true for all values of x, an equation may be true only for some values of x, or for no values of x.

Examples: $2x + 6 = 4$ is true when $x = -1$, but not when $x = 0$. The equation $x + 5 = x$ is never true, because a number is never equal to five more than itself. We say this equation has *no solution*.

1. For each equation, state whether it is **always**, **sometimes**, or **never** true. If it is always or never true, explain how you know. It may help to simplify, and to use tables, graphs, or sketches of the Lab Gear.
   a. $2x + 5 = 2x + 1$
   b. $3(x - 4) - 4(x - 3) = 0$
   c. $(x + 5)^2 = x^2 + 25$
   d. $6x - (7 - x) + 8 = 7x + 1$

2. Look at the equations in the previous problem that you decided were *sometimes* true. For each one, find a value of x that makes it true and one that makes it false. Show your work.

For each equation:
   a. Draw two graphs on the same axes: one for y = the left side of the equation, and one for y = the right side of the equation.
   b. State whether the equation is always, sometimes, or never true. Explain.

3. $.5x - 2 = .5 (x - 2)$

4. $.5x - 2 = .5 (x - 4)$

5. $.5x - 2 = x - 4$

6. $.5 (x - 2) = x - 4$

7. **Report.** Write a report about equations that are always, sometimes, or never true. Use one example of each type. Illustrate each example with a graph and a Lab Gear sketch. Be sure to include the definition of *identity,* and full explanations.

Which is Greater?

8. Which is greater, or does it depend on the value of x? Explain.

a.	$-2x$	$-2x+ 7$
b.	$6x-4$	$6x+4$
c.	$-x^2$	x^2
d.	$(-x)^2$	$-x^2$

9. Always, sometimes, or never true?
   a. $2x + 6 = 2x - 6$
   b. $2x + 6 = 2 (x + 6)$
   c. $2x + 6 = x + 6$
   d. $2x + 6 = 2 (x + 3)$

10. For each equation in #9, decide which of the two expressions is greater, if they are equal, or if it depends on the value of x.

